AP Calculus AB · Practice Worksheet · tutorinmontreal.ca

Derivative Rules

Name: ______________________________ Date: ______________

Concepts

- Average and Instantaneous Rates of Change at a Point
- Defining the Derivative and Using Derivative Notation
- Estimating the Derivative of a Function at a Point
- Connecting Differentiability and Continuity
- Applying the Power Rule
- Constant, Sum, Difference and Constant Multiple Rules (theory)
- Derivatives of cos x, sin x, e^x and ln x
- The Product Rule
- The Quotient Rule
- Derivatives of tan, cot, sec and csc (theory)

Average and Instantaneous Rates of Change at a Point

1. A camera drone rises straight up from a rooftop. Its height above the roof, in meters, is $h(t) = 40t - 5t^2$, where t is measured in seconds and $0 \leq t \leq 8$.

a) Find the average rate of change of h on the interval $[1, 3]$, with units.

b) Use the limit $\lim_{k \to 0} \dfrac{h(1+k) - h(1)}{k}$ to find the instantaneous rate of change of h at $t = 1$, with units.

Defining the Derivative and Using Derivative Notation

2. Let $f(x) = 2x^2 - 5x + 1$.

a) Use the definition $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ to find $f'(3)$.

b) If $y = f(x)$, write the value found in part (a) in Leibniz notation.

Estimating the Derivative of a Function at a Point

3. The water temperature T, in degrees Celsius, inside a solar collector is recorded every four minutes after sunrise: $T(0) = 18$, $T(4) = 26$, $T(8) = 31$, $T(12) = 34$, $T(16) = 35.5$, where t is in minutes.

a) Use the two recorded values on either side of $t = 8$ to estimate $T'(8)$. Give units.

b) Interpret your estimate in one sentence a technician would understand.

Connecting Differentiability and Continuity

4. Let $f(x) = |x - 5| + 2$.

a) Show that f is continuous at $x = 5$.

b) Using one-sided limits of the difference quotient at $x = 5$, show that $f'(5)$ does not exist.

c) State, in one sentence, what this pair of results shows about the relationship between continuity and differentiability.

Applying the Power Rule

5. Find $f'(x)$ for $f(x) = 4x^5 - 3x^{-2} + 7\sqrt{x} - 9$, and state the domain of f'.

Constant, Sum, Difference and Constant Multiple Rules

6. The functions f and g are differentiable, and $f'(4) = -2$ and $g'(4) = 5$. Let $h(x) = 3f(x) - 2g(x) + 11$. Find $h'(4)$, and name the derivative rule that justifies each step.

Derivatives of cos x, sin x, e^x and ln x

7. Find $f'(x)$ for $f(x) = 3\sin x - 4\cos x + 2e^x - 5\ln x$, and state the domain of f.

The Product Rule

8. Let $f(x) = (x^2 + 3)(2x - 5)$.

a) Find $f'(x)$ using the product rule.

b) Expand $f(x)$ first and differentiate the result, and check that the two answers agree.

The Quotient Rule

9. Find $f'(x)$ for $f(x) = \dfrac{3x - 1}{x^2 + 2}$, and simplify the numerator.

Derivatives of tan, cot, sec and csc

10. Starting from the identity $\tan x = \dfrac{\sin x}{\cos x}$, use the quotient rule to derive $\dfrac{d}{dx}\tan x$. Simplify your result to a single trigonometric function, and state the domain on which the formula holds.

Synthesis — drawing on several topics in this unit

11. Let $f(x) = x \ln x$ for $x > 0$.

a) Find $f'(x)$, naming the rule you used.

b) Write an equation of the line tangent to $y = f(x)$ at $x = e$.

c) Find the exact value of x at which the tangent line to $y = f(x)$ is horizontal.