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Chain Rule and Implicit Differentiation

Name: ____ Date: __________________

Concepts

- The Chain Rule
- Implicit Differentiation
- Differentiating Inverse Functions
- Differentiating Inverse Trigonometric Functions
- Selecting Procedures for Calculating Derivatives
- Calculating Higher-Order Derivatives

The Chain Rule

1. Differentiate each function. Leave no negative exponents in your final answer.

a) $y = \sqrt{7 - 4x^3}$

b) $g(t) = \sin^3(4t)$, and then evaluate $g'\left(\dfrac{\pi}{24}\right)$ exactly.

Implicit Differentiation

2. The curve $x^3 + y^3 = 9xy$ passes through the point $(2, 4)$.

a) Use implicit differentiation to find $\dfrac{dy}{dx}$ in terms of x and y.

b) Write an equation of the line tangent to the curve at $(2, 4)$.

Differentiating Inverse Functions

3. Let $f(x) = 2 + \sqrt{x-1}$ on the domain $[1, \infty)$, and let g be the inverse of f.

a) Explain briefly why f has an inverse on $[1, \infty)$.

b) Find $g'(4)$.

Differentiating Inverse Trigonometric Functions

4. Differentiate.

a) $y = \arctan(3x)$, and evaluate $\left.\dfrac{dy}{dx}\right|_{x=1/3}$.

b) $p(x) = \arcsin(x^2)$, and state the values of x for which your formula applies.

Selecting Procedures for Calculating Derivatives

5. Let $f(x) = \dfrac{e^{2x}}{x^2+1}$.

a) Name the differentiation rules this derivative needs, and the order in which you apply them.

b) Find $f'(x)$ and write it with the numerator fully factored.

Calculating Higher-Order Derivatives

6. Find the required derivatives.

a) For $f(x) = x^4 - 2x^3 + 7x$, find $f''(x)$ and $f'''(x)$.

b) A bead on a vertical spring has displacement $s(t) = \cos(4t)$ centimeters above its rest position, where t is in seconds. Find the acceleration $\dfrac{d^2s}{dt^2}$, and give its units.

Synthesis — drawing on several topics in this unit

7. The point $(1, 4)$ lies on the curve $2x^2 + y^2 = 18$.

a) Find $\dfrac{dy}{dx}$ in terms of x and y, and write an equation of the tangent line to the curve at $(1, 4)$.

b) Show that $\dfrac{d^2y}{dx^2} = \dfrac{-36}{y^3}$ at every point of the curve with $y \neq 0$.

c) Evaluate $\dfrac{d^2y}{dx^2}$ at $(1, 4)$ and say what its sign tells you about the curve near that point.