

Rates of Change and Related Rates

Name: _____

Date: _____

Concepts

- Interpreting the Meaning of the Derivative in Context
 - Straight-Line Motion - Position, Velocity and Acceleration
 - Rates of Change in Applied Contexts Other Than Motion
 - Introduction to Related Rates (theory)
 - Solving Related Rates Problems
 - Approximating Values Using Local Linearity and Linearization
 - Using L'Hospital's Rule for Indeterminate Forms
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Interpreting the Meaning of the Derivative in Context

1. A rooftop rain cistern is being drained through a filter. Let $W(t)$ be the volume of water in the cistern, in liters, where t is measured in hours after the valve is opened.
 - a) State the units of $W'(t)$, and explain in one sentence where those units come from.
 - b) Given that $W(3) = 418$ and $W'(3) = -26$, write one sentence interpreting $W'(3)$ in the context of the cistern, using correct units.
 - c) Explain what would be different about the cistern if $W'(3)$ were $+26$ instead.

Straight-Line Motion - Position, Velocity and Acceleration

2. A maintenance cart runs along a straight rail. Its position, in meters from the depot, is

$$s(t) = 2t^3 - 15t^2 + 24t + 5, \quad 0 \leq t \leq 5,$$

where t is in seconds.

- a) Find $v(t)$ and $a(t)$, with units.
- b) Find every time in $(0, 5)$ at which the cart is at rest.
- c) At $t = 2$ seconds, is the cart speeding up or slowing down? Justify your answer.

Rates of Change in Applied Contexts Other Than Motion

3. A greenhouse tank is filled and then drawn down over one working day. The volume of water in it is

$$V(t) = 240 + 36t - 3t^2 \text{ liters,} \quad 0 \leq t \leq 10,$$

where t is in hours after 7 a.m.

- a) Find $V'(t)$ and state its units.
- b) Find $V'(8)$ and interpret it in one sentence, with units.
- c) At what time is the volume of water greatest? Justify your answer using V' .

Introduction to Related Rates

4. Two quantities x and y are both functions of time t and satisfy

$$xy + x^2 = 40.$$

- a) Differentiate both sides with respect to t . State which differentiation rule the term xy requires, and why.
- b) Find the value of y when $x = 4$.
- c) Find $\frac{dy}{dt}$ at the instant when $x = 4$ and $\frac{dx}{dt} = -2$.
- d) A student differentiates x^2 with respect to t and writes $2x$. State the missing factor and explain in one sentence why it is there.

Solving Related Rates Problems

5. A weather balloon is inflated so that it stays spherical. Helium is pumped in at a constant rate of 36π cubic feet per minute. For a sphere of radius r , $V = \frac{4}{3}\pi r^3$ and $S = 4\pi r^2$.

- a) How fast is the radius increasing at the instant the radius is 3 feet? Include units.
- b) How fast is the surface area increasing at that same instant? Include units.

Approximating Values Using Local Linearity and Linearization

6. Let $f(x) = \sqrt[3]{x}$.

- a) Write the equation of the line tangent to the graph of f at $x = 8$.
- b) Use that tangent line to approximate $\sqrt[3]{8.06}$.
- c) Is your approximation an overestimate or an underestimate of the true value? Justify your answer using the concavity of f .

Using L'Hospital's Rule for Indeterminate Forms

7. Evaluate each limit. In each case, first show that the limit has an indeterminate form of type $\frac{0}{0}$ or $\frac{\infty}{\infty}$, so that L'Hospital's Rule may be applied.

a) $\lim_{x \rightarrow 3} \frac{x^2 - 9}{\ln(x - 2)}$

b) $\lim_{x \rightarrow \infty} \frac{4x^2 - x}{3x^2 + 5}$

Synthesis — drawing on several topics in this unit

8. A test sled is fired along a straight horizontal track. Let $s(t)$ be its position in meters from the launch point, with t in seconds. Instruments record $s(4) = 52$, $s'(4) = 9$ and $s''(4) = -2$.

- a) Interpret $s'(4)$ in one sentence in the context of the sled, with units.
- b) Use the line tangent to the graph of s at $t = 4$ to approximate $s(4.2)$, with units.
- c) Is the sled speeding up or slowing down at $t = 4$? Justify your answer.
- d) Assuming $s''(t) < 0$ for all t near 4, is your answer to (b) an overestimate or an underestimate of the true position? Justify.

Want the harder problems and the worked solutions?

The *Challenge Problems* set and the *Answer Keys* for Rates of Change and Related Rates — with the reasoning written out in full — are available at tutorinmontreal.ca.