AP Calculus AB · Practice Worksheet · tutorinmontreal.ca

Analyzing Functions with Derivatives

Name: ______________________________ Date: ______________

Concepts

- Using the Mean Value Theorem
- Extreme Value Theorem, Global versus Local Extrema, Critical Points
- Determining Intervals on Which a Function Is Increasing or Decreasing
- Using the First Derivative Test for Relative Extrema
- Using the Candidates Test for Absolute Extrema
- Determining Concavity over the Domain
- Using the Second Derivative Test to Determine Extrema (theory)
- Sketching Graphs of Functions and Their Derivatives
- Connecting a Function, Its First Derivative and Its Second Derivative
- Introduction to Optimization Problems (theory)
- Solving Optimization Problems
- Exploring Behaviors of Implicit Relations (theory)

Using the Mean Value Theorem

1. Let $f(x) = x^3 - 6x^2 + 5$ on the closed interval $[1, 4]$.

a) Verify that f satisfies the hypotheses of the Mean Value Theorem on $[1, 4]$.

b) Find the average rate of change of f over $[1, 4]$, then find every value of c in $(1, 4)$ guaranteed by the theorem.

Extreme Value Theorem, Global versus Local Extrema, Critical Points

2. Let $f(x) = x^{2/3}(x - 5)$. Find every critical point of f, and state for each one whether f' equals zero there or fails to exist there.

Determining Intervals on Which a Function Is Increasing or Decreasing

3. Let $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$. Use a sign analysis of f' to find every open interval on which f is increasing and every open interval on which f is decreasing.

Using the First Derivative Test for Relative Extrema

4. Let $f(x) = x^3 - 3x^2 - 9x + 2$. Use the First Derivative Test to locate every relative extremum of f, and give the value of f at each one.

Using the Candidates Test for Absolute Extrema

5. Let $f(x) = x^3 - 12x$ on the closed interval $[0, 3]$. Find the absolute maximum and absolute minimum values of f on that interval, and state where each occurs. Show the list of candidates you compared.

Determining Concavity over the Domain

6. Let $f(x) = x^4 - 6x^2 + 2x$. Find every open interval on which the graph of f is concave up and every open interval on which it is concave down, and give the coordinates of each point of inflection.

Using the Second Derivative Test to Determine Extrema

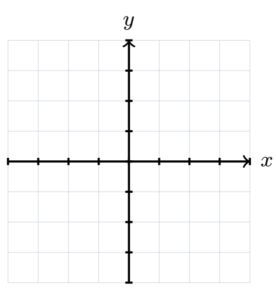
7. Let $f(x) = x^4 - 4x^3$.

a) Find the critical numbers of f and apply the Second Derivative Test at each one.

b) At any critical number where the Second Derivative Test gives no conclusion, decide what happens by another method, and say which method you used.

Sketching Graphs of Functions and Their Derivatives

8. Let $f(x) = x^3 - 3x$. On the grid, sketch the graph of $y = f(x)$ for $-2 \leq x \leq 2$. Label the coordinates of each relative extremum and of the point of inflection, and mark the x-intercepts.



Connecting a Function, Its First Derivative and Its Second Derivative

9. A function f is twice differentiable on the interval $[0, 6]$. On the open interval $(0, 3)$ both $f'(x) > 0$ and $f''(x) < 0$; on the open interval $(3, 6)$ both $f'(x) < 0$ and $f''(x) < 0$; and $f'(3) = 0$.

a) Describe the shape of the graph of f on each of the two intervals (rising or falling, and concave up or concave down).

b) Is $f(3)$ a relative maximum, a relative minimum, or neither? Justify.

c) Is $x = 3$ the location of a point of inflection? Justify.

Introduction to Optimization Problems

10. A community garden bed is to be built as a rectangle with one full side flush against the straight wall of a greenhouse. The wall needs no edging; the other three sides are edged with a total of 48 meters of timber.

a) Letting x be the length in meters of each side perpendicular to the wall, write the enclosed area A as a function of x alone.

b) State the domain of A that makes sense in this context, and explain each restriction.

Do not maximize the area.

Solving Optimization Problems

11. An open-top fermentation vat has a square base and holds exactly 32,000 cm^3. Find the dimensions that use the least stainless steel — that is, that minimize the total area of the base and the four sides — and give that minimum area. Justify that your answer really is a minimum.

Exploring Behaviors of Implicit Relations

12. The curve $x^2 + xy + y^2 = 12$ is a closed curve in the plane.

a) Use implicit differentiation to show that $\dfrac{dy}{dx} = -\dfrac{2x+y}{x+2y}$.

b) Find the coordinates of every point on the curve at which the tangent line is horizontal, and confirm that the derivative is defined at each of those points.

Synthesis — drawing on several topics in this unit

13. Let $f(x) = 2x^3 - 9x^2 + 12x + 1$ on the closed interval $[-1, 3]$.

a) Find the open intervals on which f is increasing and on which it is decreasing.

b) Locate and classify every relative extremum of f on $(-1, 3)$, using the First Derivative Test.

c) Find the open intervals of concavity and the coordinates of the point of inflection.

d) Find the absolute maximum and absolute minimum values of f on $[-1, 3]$, and state where each occurs.