AP Calculus AB · Practice Worksheet · tutorinmontreal.ca

Integration and Accumulation

Name: ______________________ Date: ____________

Concepts

- Exploring Accumulations of Change
- Approximating Areas with Riemann Sums
- Riemann Sums, Summation Notation and Definite Integral Notation
- The Fundamental Theorem of Calculus and Accumulation Functions
- Interpreting the Behavior of Accumulation Functions
- Applying Properties of Definite Integrals
- The Fundamental Theorem of Calculus and Definite Integrals
- Finding Antiderivatives and Indefinite Integrals
- Integrating Using Substitution
- Integrating Using Long Division and Completing the Square (theory)
- Selecting Techniques for Antidifferentiation (theory)

Exploring Accumulations of Change

1. Meltwater runs off a glacier into a catch basin. For $0 \leq t \leq 10$ hours it enters at a rate $r(t)$, measured in liters per hour, and the graph of r consists of two line segments: one from $(0,0)$ to $(4,60)$, and one from $(4,60)$ to $(10,0)$.

a) Using geometry, find $\displaystyle\int_0^{10} r(t)\,dt$.

b) State what this number represents about the basin, with units, and explain how the units of the answer follow from the units of r and of t.

Approximating Areas with Riemann Sums

2. Grain is loaded onto a barge. The rate of loading $R(t)$, in tons per hour, is recorded at five times during an eight-hour shift.

t (hours)	0	2	4	6	8
$R(t)$ (tons/hour)	3	5	8	12	17

a) Use a left Riemann sum with the four subintervals given by the table to approximate $\int_0^8 R(t)\,dt$.

b) Do the same with a right Riemann sum.

c) R is an increasing function on $[0, 8]$. Which of your two answers is an underestimate of $\int_0^8 R(t)\,dt$, and why?

Riemann Sums, Summation Notation and Definite Integral Notation

3. Consider the limit
$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(3 + \frac{5i}{n}\right)^2 \left(\frac{5}{n}\right).$$

a) Identify Δx, the right endpoint x_i of the ith subinterval, and the values of a and b.

b) Write the limit as a definite integral.

The Fundamental Theorem of Calculus and Accumulation Functions

4. Let $G(x) = \int_2^x \left(t^3 - 4t + 1\right)\, dt$.

a) Find $G'(x)$.

b) Find $G(2)$ and $G'(3)$.

c) Write an equation of the line tangent to the graph of $y = G(x)$ at $x = 2$.

Interpreting the Behavior of Accumulation Functions

5. The graph of a continuous function f on $[0, 8]$ consists of the line segments joining, in order, the points $(0, 0)$, $(2, 4)$, $(4, 0)$, $(6, -2)$ and $(8, 0)$. Define $G(x) = \int_0^x f(t)\,dt$ for $0 \le x \le 8$.

a) Find $G(2)$, $G(4)$ and $G(6)$.

b) Find $G(8)$.

Applying Properties of Definite Integrals

6. Suppose f and g are continuous and

$$\int_1^4 f(x)\,dx = 10, \qquad \int_4^7 f(x)\,dx = -3, \qquad \int_1^7 g(x)\,dx = 5.$$

Evaluate each of the following.

a) $\displaystyle\int_1^7 f(x)\,dx$

b) $\displaystyle\int_7^1 f(x)\,dx$

c) $\displaystyle\int_1^7 (2f(x) - 3g(x))\,dx$

d) $\displaystyle\int_4^1 f(x)\,dx + \int_1^7 g(x)\,dx$

The Fundamental Theorem of Calculus and Definite Integrals

7. Evaluate each definite integral analytically, showing the antiderivative you use.

a) $\int_1^4 \left(6x^2 - 4x + 3\right)\,dx$

b) $\int_1^9 \frac{1}{\sqrt{x}}\,dx$

Finding Antiderivatives and Indefinite Integrals

8. Find each indefinite integral.

a) $\int \left(4x^3 - \frac{6}{x^2} + 5\right)\,dx$

b) $\int \left(2e^x + \frac{3}{x}\right)\,dx$

c) $\int \left(\sec^2 x - 5\sin x\right)\,dx$

Integrating Using Substitution

9.

a) Find $\int 6x^2 \left(x^3+1\right)^3 \, dx$.

b) Evaluate $\int_0^3 x\sqrt{x^2+16}\, dx$, converting the limits of integration to the new variable.

Integrating Using Long Division and Completing the Square

10. Each integrand must be rewritten before any antidifferentiation rule applies. Find each indefinite integral, showing the rewriting step.

a) $\int \frac{2x+7}{x+3}\, dx$

b) $\int \frac{1}{x^2+6x+13}\, dx$

Selecting Techniques for Antidifferentiation

11. For each integral below, name the technique you would use — a basic antiderivative rule after algebraic rewriting, substitution, long division, or completing the square — and give the feature of the integrand that told you so. Then evaluate all four integrals.

a) $\displaystyle\int \frac{4x}{x^2+9}\,dx$

b) $\displaystyle\int \frac{x^2+2}{x^2+9}\,dx$

c) $\displaystyle\int \frac{1}{x^2-2x+10}\,dx$

d) $\displaystyle\int \frac{x-1}{\sqrt{x}}\,dx$, where $x>0$

Synthesis — drawing on several topics in this unit

12. A rainwater cistern holds 50 cubic meters of water at time $t = 0$. Water enters and leaves the cistern simultaneously, so that for $0 \le t \le 16$ hours the *net* rate at which its volume changes is $12 - 4\sqrt{t}$ cubic meters per hour. The volume in the cistern at time x hours is therefore

$$V(x) = 50 + \int_0^x \left(12 - 4\sqrt{t}\right)\, dt.$$

a) Find $V'(x)$, and state which theorem you used.

b) Find the time at which the volume of water is greatest, and justify that your answer gives the absolute maximum on $[0, 16]$.

c) Find the volume of water in the cistern at that time, and the volume at $x = 16$. Give three-decimal accuracy where the answer is not exact.

d) Using correct units, explain what $\int_9^{16} \left(12 - 4\sqrt{t}\right)\, dt$ represents in this context, and state its value.