AP Calculus AB · Practice Worksheet · tutorinmontreal.ca

Differential Equations

Name: ______________________ Date: ____________

Concepts

- Modeling Situations with Differential Equations
- Verifying Solutions for Differential Equations (theory)
- Sketching Slope Fields
- Reasoning Using Slope Fields
- Finding General Solutions Using Separation of Variables
- Finding Particular Solutions Using Initial Conditions
- Exponential Models with Differential Equations

Modeling Situations with Differential Equations

1. Translate each statement into a differential equation. Use k for a constant of proportionality and do not attempt to solve anything.

a) In a maple-sap evaporator the volume V of sap, in litres, changes at a rate proportional to the difference between V and 40 litres. Let t be measured in hours.

b) The depth h of snow on a flat roof, in centimetres, decreases at a rate inversely proportional to the square root of h. Let t be measured in days.

c) At every point (x, y) on the graph of $y = f(x)$, the slope of the tangent line equals the product of the x-coordinate and the square of the y-coordinate.

Verifying Solutions for Differential Equations

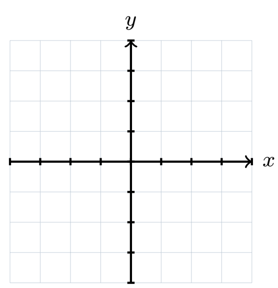
2. Consider the differential equation $\dfrac{dy}{dx} = 2y + 6$. For each function below, decide by substitution whether it is a solution, showing both sides of the equation:

$$y_1 = 5e^{2x} - 3, \qquad y_2 = -3, \qquad y_3 = 5e^{2x} + 3.$$

For the one that is not a solution, state exactly how much the two sides differ by.

Sketching Slope Fields

3. On the grid below, sketch the slope field for $\dfrac{dy}{dx} = x - y$ at the nine lattice points with $x \in \{-1, 0, 1\}$ and $y \in \{-1, 0, 1\}$: at each point draw a short segment whose slope is the value the differential equation gives there. Then state which points in the plane get a horizontal segment.

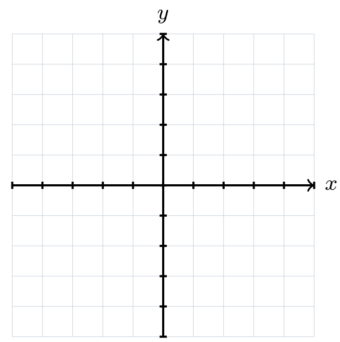


Reasoning Using Slope Fields

4. The slope field for $\dfrac{dy}{dx} = 3 - y$ is defined at every point of the plane.

a) On the grid, draw the segments at the lattice points with $x \in \{0, 1, 2\}$ and $y \in \{0, 1, 2, 3, 4\}$, then sketch the solution curve through $(0, 1)$ for $x \geq 0$.

b) State $\lim\limits_{x \to \infty} y$ for that curve and justify your answer from the slope field, not from a formula.



Finding General Solutions Using Separation of Variables

5. Find the general solution of $\dfrac{dy}{dx} = 6x^2y$. Write your answer with y isolated.

Finding Particular Solutions Using Initial Conditions

6. Solve the initial value problem $\frac{dy}{dx} = 2xy$ with $y(0) = 5$. Show how the initial condition settles the absolute value that the logarithm produces.

Exponential Models with Differential Equations

7. A tank of algae is grown for biofuel. Its mass A, in kilograms, satisfies $\frac{dA}{dt} = 0.15A$, where t is measured in days, and $A(0) = 12$ kilograms.

a) Find $A(t)$.

b) Find the exact mass after 10 days.

c) Interpret the constant 0.15 in the context of the tank, using correct units.

Synthesis — drawing on several topics in this unit

8. Consider the differential equation $\frac{dy}{dx} = \frac{2x}{y^2}$.

a) Verify that $y = \left(3x^2 + 8\right)^{1/3}$ is a solution, and state the initial condition it satisfies at $x = 0$.

b) Find the general solution by separation of variables, and show that the function in part (a) is the member of that family fixed by your initial condition.

c) Explain why the differential equation assigns no slope at all to a point where $y = 0$, and what that means for the slope field along the x-axis.