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Applications of Integration

Name: ______________________________ Date: ______________

Concepts

- Finding the Average Value of a Function on an Interval
- Connecting Position, Velocity and Acceleration Using Integrals
- Using Accumulation Functions and Definite Integrals in Applied Contexts
- Finding the Area Between Curves Expressed as Functions of x
- Finding the Area Between Curves Expressed as Functions of y
- Area Between Curves That Intersect at More Than Two Points (theory)
- Volumes with Cross Sections - Squares and Rectangles
- Volumes with Cross Sections - Triangles and Semicircles (theory)
- Volume with the Disc Method - Revolving Around the x- or y-Axis
- Volume with the Disc Method - Revolving Around Other Axes (theory)
- Volume with the Washer Method - Revolving Around the x- or y-Axis
- Volume with the Washer Method - Revolving Around Other Axes (theory)

Finding the Average Value of a Function on an Interval

1. Find the average value of $f(x) = 6x^2 - 4x$ on the interval $[1, 4]$.

Connecting Position, Velocity and Acceleration Using Integrals

2. A cart moves along a straight track. Its acceleration is $a(t) = 6t + 2$ m/s^2, its velocity at $t = 0$ is 1 m/s, and its position at $t = 0$ is $s(0) = 2$ m.

a) Find $v(t)$.

b) Find the displacement of the cart on $[0, 3]$.

c) Find $s(3)$.

Using Accumulation Functions and Definite Integrals in Applied Contexts

3. Grain is loaded into a silo at a rate of $R(t) = 20 + 6t^2$ kilograms per hour, for $0 \leq t \leq 3$ hours. The silo already holds 50 kg of grain at $t = 0$.

a) Write a definite integral giving the number of kilograms of grain added during the first 3 hours, and evaluate it.

b) How much grain is in the silo at $t = 3$?

Finding the Area Between Curves Expressed as Functions of x

4. Find the area of the region enclosed by the parabola $y = x^2$ and the line $y = 2x + 3$.

Finding the Area Between Curves Expressed as Functions of y

5. Find the area of the region enclosed by the curve $x = y^2 - 4y$ and the line $x = 5$, integrating with respect to y.

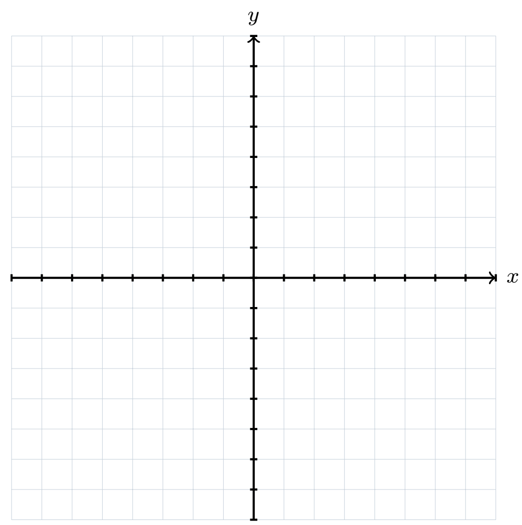
Area Between Curves That Intersect at More Than Two Points

6. The curves $y = x^3$ and $y = 4x$ meet at three points.

a) Find the three points of intersection.

b) Show that $\int_{-2}^{2} \left(4x - x^3\right)\, dx = 0$, and explain why this value is *not* the area of the region enclosed by the two curves.

c) Find the exact area enclosed by the two curves.



Volumes with Cross Sections - Squares and Rectangles

7. The base of a solid is the region bounded by the parabola $y = 4 - x^2$ and the x-axis. Every cross section perpendicular to the x-axis is a square whose side lies in the base. Find the volume of the solid.

Volumes with Cross Sections - Triangles and Semicircles

8. The base of a solid is the triangular region in the first quadrant bounded by the line $y = 6 - 2x$, the x-axis and the y-axis. Cross sections perpendicular to the x-axis are taken with one side (or the diameter) lying in the base.

a) Find the volume if every cross section is a semicircle whose diameter lies in the base.

b) Find the volume if every cross section is an equilateral triangle.

c) Which solid is larger? Justify your answer by comparing the two cross-sectional area formulas, not by comparing decimals alone.

Volume with the Disc Method - Revolving Around the x- or y-Axis

9. The region bounded by $y = x^2$, the x-axis and the line $x = 2$ is revolved about the x-axis. Find the volume of the resulting solid.

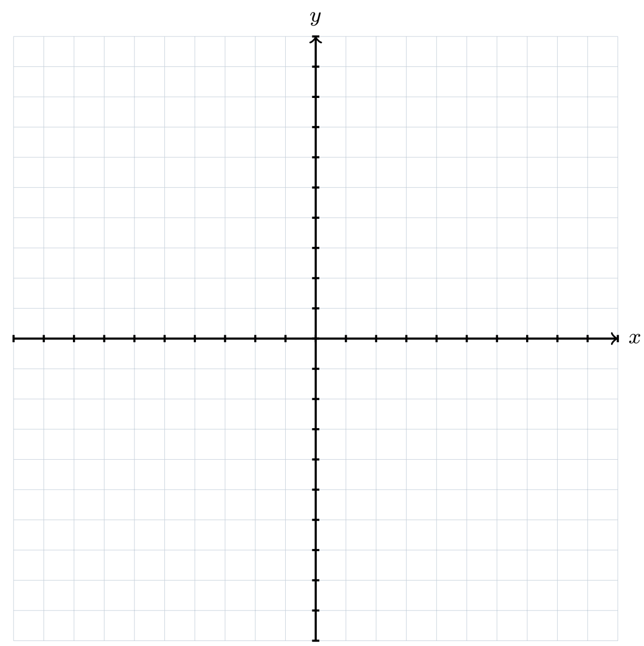
Volume with the Disc Method - Revolving Around Other Axes

10. Let R be the region bounded above by the horizontal line $y = 3$, below by the curve $y = \sqrt{x}$, and on the left by the y-axis. R is revolved about the line $y = 3$.

a) Sketch R and the axis of revolution on the grid.

b) State the radius of a typical disc as a function of x, and explain in one sentence why it is $3 - \sqrt{x}$ and not $\sqrt{x}$.

c) Find the volume of the solid.



Volume with the Washer Method - Revolving Around the x- or y-Axis

11. Let R be the region enclosed by the parabola $y = x^2$ and the line $y = 2x$. Find the volume of the solid generated when R is revolved about the x-axis.

Volume with the Washer Method - Revolving Around Other Axes

12. Let R be the region bounded by $y = x^2$, the x-axis and the line $x = 2$. R is revolved about the vertical line $x = 3$.

a) Explain why the slices are washers and why the inner radius is 1, not 0.

b) Write the outer and inner radii as functions of y, and give the y-limits.

c) Find the volume.

Synthesis — drawing on several topics in this unit

13. Let R be the region enclosed by the curve $y = 4x - x^2$ and the line $y = x$.

a) Find the coordinates of the two points where the boundaries meet, and find the area of R.

b) Find the average value, on the interval between the two x-coordinates found in part (a), of the vertical distance between the two boundaries of R.

c) R is revolved about the x-axis. Set up and evaluate an integral for the volume of the resulting solid.

d) R is the base of a solid whose cross sections perpendicular to the x-axis are squares. Find the volume of that solid.