CEGEP Calculus I · Practice Worksheet · tutorinmontreal.ca

Limits and Continuity

Name: ______________________ Date: ____________

Concepts

- Limits from a Graph and a Table
- Limit Laws and Algebraic Evaluation
- Indeterminate Forms and Algebraic Manipulation
- One-Sided Limits and Piecewise Functions (theory)
- Infinite Limits and Vertical Asymptotes
- Limits at Infinity and Horizontal Asymptotes
- Continuity at a Point
- Continuity on an Interval (theory)
- The Intermediate Value Theorem (theory)

Limits from a Graph and a Table

1. The graph of a function f consists of three pieces: a line segment rising from $(-4,-1)$ to an open circle at $(-1,2)$; a solid dot at $(-1,3)$; and a smooth curve falling from an open circle at $(-1,2)$, through $(0,1)$, to $(3,-2)$.

a) Find $\lim_{x\to -1^-} f(x)$ and $\lim_{x\to -1^+} f(x)$.

b) Does $\lim_{x\to -1} f(x)$ exist? Justify your answer from part (a).

c) State $f(-1)$, and explain in one sentence why your answer to (b) is not affected by it.

Limit Laws and Algebraic Evaluation

2. Suppose $\lim_{x\to 2} f(x) = 6$ and $\lim_{x\to 2} g(x) = -3$. Evaluate each limit, naming the limit law that justifies each step.

a) $\lim_{x\to 2}[3f(x) + 2g(x)]$

b) $\lim_{x\to 2} \dfrac{f(x)\,g(x)}{f(x) + g(x)}$

c) $\lim_{x\to 2} \sqrt{2f(x) + 4}$

Indeterminate Forms and Algebraic Manipulation

3. Evaluate each limit exactly, showing the manipulation that removes the indeterminate form.

a) $\lim_{x\to 3} \dfrac{x^2 - x - 6}{x^2 - 9}$

b) $\lim_{x\to 0} \dfrac{\sqrt{x+4} - 2}{x}$

One-Sided Limits and Piecewise Functions

4. Two functions are given.

a) For $f(x) = \dfrac{2x-10}{|x-5|}$, $x \neq 5$, find $\lim\limits_{x\to 5^-} f(x)$ and $\lim\limits_{x\to 5^+} f(x)$, and decide whether $\lim\limits_{x\to 5} f(x)$ exists.

b) For

$$g(x) = \begin{cases} x^2+1, & x<2, \\ 7-x, & x \geq 2, \end{cases}$$

find $\lim\limits_{x\to 2^-} g(x)$, $\lim\limits_{x\to 2^+} g(x)$, and $\lim\limits_{x\to 2} g(x)$ if it exists.

c) State the condition on the one-sided limits under which a two-sided limit exists.

Infinite Limits and Vertical Asymptotes

5. Let $f(x) = \dfrac{x+1}{x^2 - x - 6}$.

a) Find the equations of the vertical asymptotes of f, and say what makes each one an asymptote rather than a hole.

b) Evaluate the four one-sided limits at those two values of x.

Limits at Infinity and Horizontal Asymptotes

6. Evaluate each limit, then state the horizontal asymptotes of the function involved.

a) $\displaystyle\lim_{x\to\infty} \frac{3x^2 - 5x}{2x^2 + 7}$

b) $\displaystyle\lim_{x\to\infty} \frac{4x+1}{x^3 - 2}$

c) $\displaystyle\lim_{x\to\infty} \frac{x^3+1}{2x^2 - x}$

7. Let $f(x) = \dfrac{\sqrt{4x^2 + 3x}}{x + 1}$.

a) Evaluate $\lim_{x \to \infty} f(x)$.

b) Evaluate $\lim_{x \to -\infty} f(x)$.

c) State every horizontal asymptote of f, and explain in one sentence why the two answers differ in sign.

Continuity at a Point

8. A function is defined by

$$f(x) = \begin{cases} \dfrac{x^2 - x - 6}{x - 3}, & x \neq 3, \\[2ex] 4, & x = 3. \end{cases}$$

a) State the three conditions required for f to be continuous at $x = 3$, and check each one.

b) Classify the discontinuity.

c) Change one number in the definition so that f becomes continuous at $x = 3$.

Continuity on an Interval

9. Let $f(x) = \dfrac{\sqrt{5-x}}{x^2 - x - 2}$. Determine the largest intervals on which f is continuous, naming the results about continuity that you use. Pay particular attention to the point $x = 5$.

The Intermediate Value Theorem

10. Let $p(x) = x^3 - 4x + 1$.

a) Show that the equation $p(x) = 0$ has a solution in the interval $(0, 1)$, naming the theorem you use and checking each of its hypotheses.

b) By evaluating p twice more, locate a solution inside an interval of length 0.25.

c) Does your argument show that the solution is unique? Justify your answer.

Synthesis — drawing on several topics in this set

11. A holding tank at a desalination plant is monitored from $t = 0$ hours. The salt concentration, in mg/L, is modelled by

$$c(t) = \begin{cases} \dfrac{t^2 - 9}{t - 3}, & 0 \leq t < 3, \\ a, & t = 3, \\ \dfrac{18t + b}{t + 3}, & t > 3, \end{cases}$$

where a and b are constants.

a) Find the values of a and b that make c continuous at $t = 3$.

b) With those values, evaluate $\lim_{t \to \infty} c(t)$ and interpret the result in context, with units.

c) Does the concentration ever actually reach the value found in (b)? Justify algebraically.

Want the harder problems and the worked solutions?

The *Challenge Problems* set and the *Answer Keys* for Limits and Continuity — with the reasoning written out in full — are available at **tutorinmontreal.ca**.