CEGEP Calculus I · Practice Worksheet · tutorinmontreal.ca

Limits with Trig, Exponential and Log Functions

Name: ____ Date: __________________

Concepts

- Limits of Trigonometric Functions and the Squeeze Theorem
- Limits of Exponential and Logarithmic Functions
- Limits at Infinity with Exponentials, Logarithms and Arctangent
- Continuity of Trigonometric, Exponential and Logarithmic Functions (theory)

Limits of Trigonometric Functions and the Squeeze Theorem

1. Each limit gives $\dfrac{0}{0}$ on direct substitution. Rewrite the quotient with a trigonometric identity, cancel, and then substitute.

a) $\displaystyle\lim_{x\to 0} \frac{1-\cos x}{\sin^2 x}$

b) $\displaystyle\lim_{x\to \pi/2} \frac{\sin 2x}{\cos x}$

c) $\displaystyle\lim_{x\to 3\pi/4} \frac{1+\tan x}{\sin x+\cos x}$

2. Use the Squeeze Theorem for each limit. Write the inequality you squeeze with, and say why it holds.

a) $\lim_{x\to 0} x^2\left(2-\cos\dfrac{1}{x}\right)$

b) $\lim_{x\to 0^+} \sqrt{x}\, e^{\sin(1/x)}$

Limits of Exponential and Logarithmic Functions

3. Find each limit. Where the answer is ∞ or $-\infty$, say which, and decide it from the sign of the quantity that approaches 0.

a) $\lim_{x\to 2^-} e^{1/(4-x^2)}$ and $\lim_{x\to 2^+} e^{1/(4-x^2)}$

b) Is the line $x=2$ a vertical asymptote of $y=e^{1/(4-x^2)}$? Is $x=-2$?

c) $\lim_{x\to 0^+} \ln(\sin x)$

d) $\lim_{x\to 1^-} \dfrac{2}{1-e^{x-1}}$

4. Each limit is indeterminate on direct substitution. Use a law of logarithms or factor an expression in e^x (or in 3^x), then evaluate.

a) $\displaystyle \lim_{x \to 3^+} \left[\ln(x^2 - 9) - \ln(2x - 6)\right]$

b) $\displaystyle \lim_{x \to \ln 2} \frac{e^{2x} - e^x - 2}{e^x - 2}$

c) $\displaystyle \lim_{x \to 0} \frac{27^x - 1}{9^x - 1}$

d) $\displaystyle \lim_{x \to 1} \frac{\log_3\left(x^4\right)}{\log_9 x}$

Limits at Infinity with Exponentials, Logarithms and Arctangent

5. Some of these limits exist and some do not. Evaluate each one that exists, and explain why each of the others does not.

a) $\lim_{x \to -\infty} \cos(e^x)$

b) $\lim_{x \to \infty} \cos(e^x)$

c) $\lim_{x \to \infty} \arctan\left(x^2 - 5x\right)$

d) $\lim_{x \to \infty} \dfrac{5}{3 - \ln x}$

6. Evaluate each limit. For a quotient of exponentials, divide the numerator and the denominator by the exponential that dominates in the direction x is going.

a) $\displaystyle\lim_{x\to\infty} \frac{4e^{2x} - e^{x}}{3 + 2e^{2x}}$

b) $\displaystyle\lim_{x\to\infty} \frac{5^{x} - 4^{x}}{5^{x+1} + 2\cdot 4^{x}}$

c) $\displaystyle\lim_{x\to-\infty} \frac{4e^{-2x} + 1}{3 - e^{-2x}}$

d) $\displaystyle\lim_{x\to\infty} \left[\ln(2x+1) - \tfrac{1}{2}\ln\left(x^2+3\right)\right]$

Continuity of Trigonometric, Exponential and Logarithmic Functions

7. Find the largest intervals on which each function is continuous. For each one, name the facts about continuous functions that justify your answer.

a) $f(x) = \arcsin(\ln x)$

b) $g(x) = \dfrac{\sqrt{x+2}}{e^x - 1}$

c) $h(x) = \ln(\cos x)$, for $-2\pi < x < 2\pi$

Synthesis — drawing on several topics in this set

8. Let

$$f(x) = \begin{cases} \dfrac{e^{2x} - e^{x} - 6}{e^{x} - 3}, & x < \ln 3, \\ k + 6e^{-x}, & x \geq \ln 3, \end{cases}$$

where k is a constant.

a) Find the value of k that makes f continuous at $x = \ln 3$.

b) With that value of k, find $\lim\limits_{x \to -\infty} f(x)$ and $\lim\limits_{x \to \infty} f(x)$, and state the horizontal asymptotes of f.

c) Explain why f is then continuous at every real number.