CEGEP Calculus I · Practice Worksheet · tutorinmontreal.ca

The Derivative

Name: ______________________________ Date: ______________

Concepts

- Average and Instantaneous Rate of Change
- The Definition of the Derivative
- The Derivative as a Function (theory)
- Differentiability and Continuity (theory)
- The Equation of the Tangent Line

Average and Instantaneous Rate of Change

1. A construction hoist climbs the outside of a tower. Its height above the loading platform, in metres, is

$$h(t) = \frac{1}{2}t^2 + 3t, \qquad 0 \le t \le 10,$$

where t is measured in seconds.

a) Find the average rate of change of h on the interval $[2, 6]$, with units.

b) Use the limit $\lim_{k \to 0} \dfrac{h(2+k) - h(2)}{k}$ to find the instantaneous rate of change of h at $t = 2$, with units.

c) The two answers are different. Which one is the reading on the hoist's speedometer at the instant $t = 2$, and why?

The Definition of the Derivative

2. Let $f(x) = \sqrt{3x+4}$.

a) Use the definition $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ to find $f'(x)$.

b) State the domain of f and the domain of f' in interval notation, and say why they are not the same.

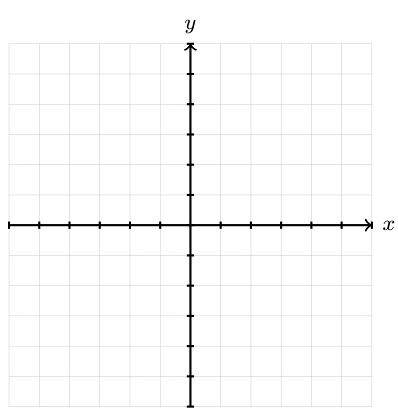
The Derivative as a Function

3. Let $f(x) = \frac{1}{3}x^3 - x$.

a) Use the definition of the derivative to find the *function* f'.

b) State the domain of f', and sketch $y = f'(x)$ on the grid below.

c) Using your sketch, state the interval on which the values of f are falling as x increases, and the values of x at which f has a horizontal tangent. Justify each answer by referring to the sign or the value of f'.



Differentiability and Continuity

4. Let $f(x) = |x^2 - 9|$.

a) Show that f is continuous at $x = 3$.

b) Compute $\displaystyle\lim_{x\to 3^-} \frac{f(x) - f(3)}{x - 3}$ and $\displaystyle\lim_{x\to 3^+} \frac{f(x) - f(3)}{x - 3}$, and deduce whether $f'(3)$ exists.

c) What do (a) and (b) together show about the relationship between continuity and differentiability?

The Equation of the Tangent Line

5. Let $f(x) = \dfrac{4}{x}$.

a) Use the definition of the derivative to show that $f'(x) = -\dfrac{4}{x^2}$ for $x \neq 0$.

b) Find the equation of the tangent line to the graph of f at the point where $x = 2$.

c) Verify algebraically that this tangent meets the graph of f at that point only.

Synthesis — drawing on several topics in this set

6. A reservoir is drawn down over a dry summer. Its volume, in millions of cubic metres, is

$$V(t) = \tfrac{1}{2}(12 - t)^2, \qquad 0 \le t \le 12,$$

where t is the number of days since drawdown began.

a) Find the average rate of change of V on $[0, 4]$, with units, and say what its sign means.

b) Use the definition of the derivative to find $V'(t)$, and evaluate $V'(4)$ with units.

c) Write the equation of the tangent line to the graph of V at $t = 4$, and use it to estimate $V(5)$.

d) Compare your estimate with the exact value of $V(5)$, and say whether the tangent line over-estimates or under-estimates. Then state the day on which the tangent line predicts the reservoir is empty, and the day it is actually empty.