CEGEP Calculus I · Practice Worksheet · tutorinmontreal.ca

Differentiation Rules

Name: ______________________________________ Date: _________________

Concepts

- Basic Differentiation Formulas
- The Product Rule
- The Quotient Rule
- The Chain Rule
- Implicit Differentiation
- Higher-Order Derivatives (theory)

Basic Differentiation Formulas

1. Differentiate each function. Rewrite first where that avoids a rule you have not needed yet, and state any restriction on the domain.

a) $f(x) = 5x^4 - 3x^2 + 7x - 2$

b) $g(x) = \dfrac{4}{x^3} + \sqrt[3]{x}$

c) $h(x) = \dfrac{2x^3 - x + 6}{x^2}$

The Product Rule

2. Let $f(x) = (3x^2 - 5)(x^3 + 2x)$.

a) Find $f'(x)$ using the product rule, and simplify.

b) Expand $f(x)$ first and differentiate the resulting polynomial. Confirm that the two answers agree.

The Quotient Rule

3. Let $f(x) = \dfrac{2x - 1}{x^2 + 3}$.

a) Find $f'(x)$ and simplify the numerator.

b) Find the equation of the tangent line to $y = f(x)$ at $x = 1$.

The Chain Rule

4. Differentiate, simplifying where a common factor allows it.

a) $f(x) = \left(2x^3 - 5x\right)^4$

b) $g(x) = \sqrt{9 - x^2}$ (state the set on which g' exists)

Implicit Differentiation

5. The curve C has equation $x^2 + 3xy - y^2 = 9$.

a) Verify that the point $(3, 0)$ lies on C.

b) Find $\dfrac{dy}{dx}$ in terms of x and y.

c) Find the equation of the tangent to C at $(3, 0)$.

Higher-Order Derivatives

6. An ore trolley runs on a straight track. Its position, in metres from a marker on the track, is modelled by $s(t) = t^3 - 6t^2 + 9t$ for $0 \leq t \leq 4$, with t in seconds.

a) Find the velocity $v(t) = s'(t)$ and the acceleration $a(t) = s''(t)$, with units.

b) Find the time at which the acceleration is zero, and state what is happening to the velocity at that instant.

c) A student says "the acceleration is zero at that time, so the trolley is momentarily at rest". Explain why this is wrong, and give the times in $[0, 4]$ at which the trolley really is at rest.

Synthesis — drawing on several topics in this set

7. A directional antenna sits at the top of a mast, 3 m above level ground. A receiver placed on the ground at a horizontal distance of d metres from the foot of the mast reports a normalised line-of-sight reading — the horizontal separation divided by the straight-line separation — modelled by

$$R(d) = \frac{d}{\sqrt{d^2+9}}\,, \qquad d \geq 0.$$

a) Find $R'(d)$ using the quotient rule together with the chain rule, and simplify to a single term.

b) Evaluate $R'(4)$ to three decimal places, with units.

c) Obtain $R'(d)$ a second way, by writing $R(d) = d\left(d^2+9\right)^{-1/2}$ and using the product rule. Then use the sign of R' to describe how the reading behaves as d increases.

Want the harder problems and the worked solutions?
The *Challenge Problems* set and the *Answer Keys* for Differentiation Rules — with the reasoning written out in full — are available at **tutorinmontreal.ca**.