

CEGEP CALCULUS I · PRACTICE WORKSHEET · TUTORINMONTREAL.CA

Indeterminate Forms and L'Hospital's Rule

Name: _____

Date: _____

Concepts

- Recognizing an Indeterminate Form
 - L'Hospital's Rule for Quotient Forms
 - Choosing Between Algebra and L'Hospital's Rule (theory)
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Recognizing an Indeterminate Form

1. For each limit below, carry out direct substitution and write down the form it produces. State whether that form is indeterminate. If it is not indeterminate, give the value of the limit (or write "does not exist", with a reason).

a) $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 - 1}$

b) $\lim_{x \rightarrow 2^-} \frac{x + 4}{x - 2}$

c) $\lim_{x \rightarrow \infty} \frac{3x^2 - x}{5x^2 + 7}$

d) $\lim_{x \rightarrow 0} \frac{\cos x}{x^2}$

e) $\lim_{x \rightarrow \infty} \frac{\ln x}{e^x}$

f) $\lim_{x \rightarrow 0} \frac{x^2}{\sin x + 3}$

L'Hospital's Rule for Quotient Forms

2. Evaluate each limit with L'Hospital's Rule. State the form you get from substitution before each application of the rule, so that it is clear the rule is allowed each time.

a) $\lim_{x \rightarrow 0} \frac{e^{3x} - 3x - 1}{x^2}$

b) $\lim_{x \rightarrow \infty} \frac{\ln(1 + 5x)}{\ln(1 + 2x)}$

c) $\lim_{x \rightarrow \pi/2} \frac{\cos x}{x - \frac{\pi}{2}}$

Choosing Between Algebra and L'Hospital's Rule

3. For each limit below, decide whether it is settled more efficiently by algebraic manipulation, by L'Hospital's Rule, or equally well by either. Justify the choice in one sentence — saying what the rejected method would cost you — and then evaluate the limit by the method you chose.

a) $\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4}$

b) $\lim_{x \rightarrow \infty} \frac{\sqrt{9x^2 + 4x}}{2x + 7}$

c) $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{\sin 3x}$

Synthesis — drawing on several topics in this set

4. Let

$$g(x) = \frac{\sqrt{x+9} - 3}{x^2 + 2x}.$$

- a) State the domain of g , then evaluate $\lim_{x \rightarrow 0} g(x)$ twice — once algebraically and once with L'Hospital's Rule — and confirm the two agree.
- b) Determine $\lim_{x \rightarrow -2} g(x)$. Explain first why L'Hospital's Rule must not be used here, and say what wrong answer it would have produced.
- c) Determine $\lim_{x \rightarrow \infty} g(x)$, naming the form and the method you chose.

Want the harder problems and the worked solutions?

The *Challenge Problems* set and the *Answer Keys* for Indeterminate Forms and L'Hospital's Rule — with the reasoning written out in full — are available at tutorinmontreal.ca.