

Analysis of Functions and Curve Sketching

Name: _____

Date: _____

Concepts

- Domain, Zeros and Intercepts (theory)
 - Locating Vertical and Horizontal Asymptotes
 - Increasing and Decreasing Intervals
 - Relative and Absolute Extrema
 - Concavity and Points of Inflection
 - Sketching a Complete Curve
 - Rolle's Theorem and the Mean Value Theorem (theory)
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Domain, Zeros and Intercepts

1. Determine the domain, the zeros and the y -intercept of each function. Where a feature does not exist, say so and explain why.

a) $f(x) = \frac{x^3 - 4x}{x^2 - 2x - 3}$

b) $h(x) = \sqrt{x+1} \ln x$

Locating Vertical and Horizontal Asymptotes

2. Let $f(x) = \frac{3x^2 - 12}{x^2 - x - 6}$. Find every vertical and every horizontal asymptote of the graph of f , and justify each one with a limit. Identify the point where the denominator is zero but no vertical asymptote occurs, and give its coordinates.

Increasing and Decreasing Intervals

3. Let $f(x) = x^2 e^{-x}$. Find f' , then determine every interval on which f is increasing and every interval on which it is decreasing, supporting your answer with a sign analysis of f' .

Relative and Absolute Extrema

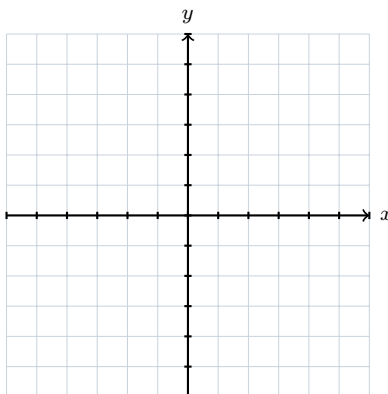
4. Let $f(x) = 2x^3 - 9x^2 + 12x$ on the closed interval $[0, 3]$.
- a) Find the critical numbers of f and classify each one with the first derivative test.
 - b) Find the absolute maximum and the absolute minimum of f on $[0, 3]$, stating where each occurs.

Concavity and Points of Inflection

5. Let $f(x) = x^4 - 4x^3 + 5$. Determine the intervals on which the graph of f is concave up and those on which it is concave down, and give the coordinates of every point of inflection.

Sketching a Complete Curve

6. Carry out a complete study of $f(x) = \frac{x^2}{x^2 - 4}$ — domain, intercepts, symmetry, vertical and horizontal asymptotes with the relevant limits, intervals of increase and decrease with any relative extrema, and intervals of concavity with any points of inflection — then sketch the curve on the grid, labelling each asymptote.



Rolle's Theorem and the Mean Value Theorem

7. Name each hypothesis as you check it.

- a) Verify that $f(x) = x^3 - 4x$ satisfies the hypotheses of Rolle's Theorem on $[-2, 2]$, then find every value of c the theorem guarantees.
- b) Verify that $g(x) = \sqrt{x}$ satisfies the hypotheses of the Mean Value Theorem on $[1, 9]$, then find every value of c the theorem guarantees.

Synthesis — drawing on several topics in this set

8. Let $f(x) = \frac{6x}{x^2 + 3}$. Carry out a full analysis:

- a) domain, zeros, y -intercept and symmetry;
- b) every vertical and horizontal asymptote, justified by a limit;
- c) intervals of increase and decrease, and every relative extremum;
- d) the absolute maximum and absolute minimum values of f on $\mathbb{R}$, with a reason — note that the Extreme Value Theorem does not apply here;
- e) intervals of concavity and every point of inflection;
- f) a short description of the resulting shape.

Want the harder problems and the worked solutions?

The *Challenge Problems* set and the *Answer Keys* for Analysis of Functions and Curve Sketching — with the reasoning written out in full — are available at tutorinmontreal.ca.