CEGEP Calculus I · Practice Worksheet · tutorinmontreal.ca

Related Rates

Name: _____ Date: _____

Concepts

- Setting Up a Related Rates Relation
- Related Rates in Geometric Figures
- Related Rates in Motion Problems (theory)
- Interpreting a Related Rates Answer (theory)

Setting Up a Related Rates Relation

1. A sealed bicycle pump holds a fixed quantity of air. While the plunger is pushed in slowly, the pressure P (in kilopascals) and the volume V (in cubic centimetres) of the trapped air satisfy

$$PV = 7200.$$

At the instant the trapped air occupies 60 cm^3, the volume is decreasing at 8 cm^3/s.

a) Name the two quantities that vary with time, and state which rate is given and which is wanted, using Leibniz notation.

b) Differentiate the relation with respect to time.

c) Find the rate of change of the pressure at that instant, with units.

Related Rates in Geometric Figures

2. Road salt is poured into a hopper shaped like an inverted right circular cone. The hopper is 6 metres deep and its circular top has radius 3 metres. Salt enters at a constant 2 m^3/min, and the pile in the hopper always fills the cone up to a depth h.

Recall $V = \frac{1}{3}\pi r^2 h$ for a cone of base radius r and height h.

a) Express V in terms of h alone. Explain why the relation $r = h/2$ may be substituted before differentiating, while $h = 4$ may not.

b) Find the rate at which the depth of salt is rising when $h = 4$ metres. Give an exact value and a decimal to three places, with units.

Related Rates in Motion Problems

3. Two straight service roads leave a depot, one due east and one due north. A van drives away from the depot along the east road at a constant 60 km/h, and a truck drives away along the north road at a constant 45 km/h.

Consider the instant at which the van is 12 km from the depot and the truck is 9 km from the depot.

a) Let x and y be the two distances from the depot and z the distance between the vehicles. Write the relation connecting them and differentiate it with respect to time.

b) Find $\dfrac{dz}{dt}$ at that instant, with units, and say in one sentence what is happening to the gap between the vehicles.

Interpreting a Related Rates Answer

4. A settling tank is being drained. An engineer models the tank correctly and, for one particular instant, obtains

$$\frac{dV}{dt} = -0.25 \text{ m}^3/\text{min}, \qquad \frac{dh}{dt} = -0.05 \text{ m/min}, \qquad \frac{dA}{dt} = 0 \text{ m}^2/\text{min},$$

where V is the volume of liquid, h is the depth and A is the area of the free surface of the liquid. No further computation of derivatives is needed below.

a) Write one sentence for each rate, saying what it tells you about the tank at that instant, with units, and whether the quantity is growing or shrinking.

b) These quantities satisfy $\dfrac{dV}{dt} = A\,\dfrac{dh}{dt}$. Use this to state the free-surface area at that instant, and explain why $\dfrac{dA}{dt} = 0$ does not mean the surface has area zero. What does it say about the shape of the tank at that depth?

c) A colleague concludes: "$\dfrac{dh}{dt}$ is only -0.05, so the tank will still be draining an hour from now." Explain why the computed rate does not justify that claim.

Synthesis — drawing on several topics in this set

5. A water trough is 4 metres long. Its cross-section is an isosceles triangle standing point down: 1 metre deep, and 1.2 metres across at the top. Water runs in at a constant 0.06 m^3/min. Let h be the depth of water, in metres.

a) Express the volume of water in terms of h alone, stating which substitution is legitimate before differentiating and why.

b) Find the rate at which the water level is rising when $h = 0.5$ metres, with units.

c) Without computing a second value, explain whether the level rises faster or more slowly as the trough continues to fill, and justify your answer from the relation you differentiated.