CEGEP Calculus I · Practice Worksheet · tutorinmontreal.ca

Optimization

Name: ______________________________ Date: ______________

Concepts

- Expressing a Quantity as a Function of One Variable
- Optimization on a Closed Interval
- Optimization Without a Closed Interval
- Justifying and Interpreting an Optimum (theory)

Expressing a Quantity as a Function of One Variable

1. A depot manager has 180 m of snow fencing to enclose a rectangular salt storage yard. One full side of the yard is the straight wall of an existing garage and needs no fencing; the fencing is used for the other three sides. Let x be the length, in metres, of each of the two sides perpendicular to the garage wall.

a) Express the enclosed area A as a function of x alone.

b) State the domain that the physical situation allows, and say in one sentence what goes wrong at each excluded value.

Do not differentiate anything — this question asks only for the model.

Optimization on a Closed Interval

2. During a 6-hour maintenance window the electrical power drawn by a data centre's cooling loop is modelled by

$$P(t) = t^3 - 9t^2 + 24t + 5 \qquad \text{kilowatts,}$$

where t is the number of hours since the window opened, $0 \le t \le 6$. Find the absolute maximum and the absolute minimum power drawn during the window, and state the time at which each occurs.

Optimization Without a Closed Interval

3. A rooftop garden needs a galvanised planter box with a square base and *no lid*, holding exactly $32\,000$ cm^3 of soil. Let x be the side of the square base, in centimetres.

a) Express the total area of sheet metal used as a function of x, and state the domain.

b) Find the dimensions that use the least metal, and the amount used.

c) The domain is not a closed interval, so the closed-interval method does not apply. Justify that your answer really is a global minimum.

Justifying and Interpreting an Optimum

4. A workshop that builds sea kayaks finds that when it makes x kayaks per week its average cost per kayak is

$$A(x) = 0.5x + \frac{800}{x} + 120 \qquad \text{dollars per kayak,} \qquad x > 0.$$

A student differentiates, solves $A'(x) = 0$, and writes down the single line "the minimum is 40".

a) Reproduce the calculation, then rewrite that conclusion as a full sentence naming what 40 is, what the minimum value is, and the units of each.

b) Justify, using the second derivative, that this really is a minimum, and explain why the test settles the question over the whole domain $x > 0$ without any interval being tested.

c) The owner reads the note and concludes "so building 40 kayaks a week minimises our costs". Explain precisely what is wrong with that reading.

Synthesis — drawing on several topics in this set

5. A park authority is building a rectangular observation deck against a straight cliff face. The cliff forms one side and needs no railing; 60 m of railing is available for the other three sides. A safety bylaw requires the deck to project at least 18 m out from the cliff and at most 25 m.

a) Let x be the projection from the cliff, in metres. Express the deck area A as a function of x, give the domain the geometry allows, and give the interval the bylaw allows.

b) Using the closed-interval method on the bylaw's interval, find the largest and the smallest deck the authority can build, with dimensions.

c) Interpret the result: state the answer in a sentence with units, and say how much deck area the bylaw costs compared with the best deck the railing alone would allow.