CEGEP Calculus II · Practice Worksheet · tutorinmontreal.ca

Antiderivatives and the Indefinite Integral

Name: _____ Date: _____

Concepts

- Antiderivatives and the Constant of Integration
- Basic Integration Formulas
- Integration by Substitution
- Substitution with Exponential and Trigonometric Integrands (theory)
- Recovering a Function from Its Rate of Change (theory)

Antiderivatives and the Constant of Integration

1. Find the most general antiderivative of

$$f(x) = 6x^2 - \frac{4}{x^3} + \sqrt[3]{x},$$

then verify your result by differentiating it. State the values of x for which your answer is valid.

Basic Integration Formulas

2. Evaluate each indefinite integral, and state where the answer is valid.

a) $\displaystyle\int \frac{2x^3 - 5\sqrt{x} + 7}{x^2}\,dx$

b) $\displaystyle\int \left(3e^x + \sec^2 x - \frac{2}{x}\right)\,dx$

Integration by Substitution

3. Evaluate, stating the substitution you use and checking each answer by differentiation.

a) $\int x^2\sqrt{x^3+5}\,dx$

b) $\int \dfrac{6x}{(3x^2+4)^2}\,dx$

Substitution with Exponential and Trigonometric Integrands

4. Evaluate each integral. In each case state the substitution, and give any restriction on x that your answer needs.

a) $\int xe^{-x^2}\,dx$

b) $\int \dfrac{\cos(\ln x)}{x}\,dx$

c) $\int \dfrac{e^{2x}}{e^{2x}+1}\,dx$

Recovering a Function from Its Rate of Change

5. A survey drone rises vertically from its launch pad. Its vertical velocity t seconds after launch is

$$v(t) = 12 - 6\sqrt{t} \qquad \text{metres per second,} \qquad 0 \le t \le 9,$$

and at the moment of launch the drone sits 2 metres above the ground.

a) Find the height $h(t)$ of the drone above the ground.

b) The drone stops climbing when its vertical velocity first reaches zero. How high is it then?

Synthesis — drawing on several topics in this set

6. Rainwater is fed into a storage cistern at a rate

$$\frac{dV}{dt} = \frac{60t}{\sqrt{t^2 + 9}} \qquad \text{litres per hour,}$$

where t is the number of hours since the storm began. The cistern already holds 100 litres when the storm starts.

a) Find $V(t)$, the volume of water in the cistern.

b) How much water is in the cistern four hours into the storm, and how much of that arrived during the storm? Give both answers with units.

Want the harder problems and the worked solutions?

The *Challenge Problems* set and the *Answer Keys* for Antiderivatives and the Indefinite Integral — with the reasoning written out in full — are available at **tutorinmontreal.ca**.