CEGEP Calculus II · Practice Worksheet · tutorinmontreal.ca

The Definite Integral and the Fundamental Theorem

Name: ______________________________ Date: ______________

Concepts

- Area Under a Curve and Riemann Sums
- The Definite Integral as a Limit of a Riemann Sum
- Properties of the Definite Integral (theory)
- The Fundamental Theorem, Part One
- The Fundamental Theorem, Part Two
- Average Value and the Mean Value Theorem for Integrals (theory)

Area Under a Curve and Riemann Sums

1. Let $f(x) = x^2 + 1$ on $[0, 2]$.

a) Estimate the area under the graph of f over $[0, 2]$ using the right-endpoint Riemann sum R_4 (four subintervals of equal width).

b) The exact area is $\frac{14}{3}$. State whether R_4 over- or under-estimates it, and justify your answer from the shape of the graph rather than from the two numbers.

The Definite Integral as a Limit of a Riemann Sum

2. Evaluate $\int_0^3 (x^2+1)\,dx$ *directly from the definition*, as the limit of right-endpoint Riemann sums. You may use $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ and $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

Properties of the Definite Integral

3. Suppose f and g are continuous and

$$\int_1^5 f(x)\,dx = 8, \qquad \int_5^9 f(x)\,dx = 6, \qquad \int_1^5 g(x)\,dx = -3.$$

a) Find $\displaystyle\int_1^5 (2f(x) - 3g(x))\,dx$.

b) Find $\displaystyle\int_9^1 f(x)\,dx$.

c) Explain why the data above is *not* enough to determine $\displaystyle\int_1^5 f(x)g(x)\,dx$, and support the explanation with an example.

The Fundamental Theorem, Part One

4. Use the Fundamental Theorem of Calculus, Part One, to differentiate each accumulation function. Do not attempt to evaluate the integrals.

a) $F(x) = \displaystyle\int_3^x \frac{t}{1+t^4}\,dt$. Give $F'(x)$, and state the value of $F(3)$.

b) $G(x) = \displaystyle\int_x^5 e^{-t^2}\,dt$.

c) $H(x) = \displaystyle\int_0^{x^3} \sqrt{1+t^2}\,dt$.

The Fundamental Theorem, Part Two

5. Evaluate each definite integral with an antiderivative, showing the antiderivative used.

a) $\displaystyle\int_1^4 \left(3\sqrt{x} - \frac{2}{x^2}\right) dx$

b) $\displaystyle\int_1^e \frac{(\ln x)^2}{x}\,dx$

Average Value and the Mean Value Theorem for Integrals

6. Over one twelve-hour daytime period the temperature inside a greenhouse is modelled by

$$T(t) = 18 + 6\sin\left(\frac{\pi t}{12}\right) \text{ degrees Celsius,} \qquad 0 \le t \le 12,$$

where t is the number of hours after opening. A calculator is needed for the decimals requested below.

a) Find the average temperature over the twelve hours, exactly and to three decimal places.

b) The Mean Value Theorem for Integrals guarantees at least one time c at which the temperature equals that average. State why its hypothesis holds here, then find every such c in $[0, 12]$, to three decimal places.

Synthesis — drawing on several topics in this set

7. Let $I = \int_0^4 (x^2 + 2)\, dx$.

a) Write I as a limit of right-endpoint Riemann sums and evaluate that limit, using $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

b) Evaluate I again with the Fundamental Theorem, Part Two, and confirm the two agree.

c) Using the properties of the definite integral — not a fresh computation — state the values of $\int_4^0 (x^2 + 2)\, dx$ and $\int_0^4 (3x^2 + 6)\, dx$.

Want the harder problems and the worked solutions?

The *Challenge Problems* set and the *Answer Keys* for The Definite Integral and the Fundamental Theorem — with the reasoning written out in full — are available at **tutorinmontreal.ca**.