

Techniques of Integration

Name: _____

Date: _____

Concepts

- Integration by Parts
 - Repeated Integration by Parts
 - Trigonometric Integrals
 - Trigonometric Substitution
 - Partial Fractions
 - Choosing an Integration Technique (theory)
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Integration by Parts

1. Evaluate each integral by parts, stating your choice of u and dv before you begin.

a) $\int x e^{-x} dx$

b) $\int_1^e x^2 \ln x dx$

Repeated Integration by Parts

2. Let $F(x) = \int x^2 \cos x \, dx$.

a) Find $F(x)$, applying integration by parts twice.

b) Use your antiderivative to evaluate $\int_0^{\pi/2} x^2 \cos x \, dx$.

Trigonometric Integrals

3. Evaluate, using an appropriate identity in each case.

a) $\int \sin^3 x \cos^2 x \, dx$

b) $\int_0^{\pi/4} \sin^2(2x) \, dx$

State which feature of the integrand told you which identity to reach for.

Trigonometric Substitution

4. Evaluate each integral by a trigonometric substitution. State the substitution, and describe the right triangle you use to return to x .

a) $\int \sqrt{16 - x^2} \, dx, \quad \text{for } -4 < x < 4$

b) $\int \frac{dx}{(x^2 + 9)^{3/2}}$

Partial Fractions

5. Decompose each integrand into partial fractions, then integrate.

a) $\int \frac{5x - 1}{x^2 - x - 2} dx$

b) $\int \frac{x + 5}{x(x + 1)^2} dx$

Choosing an Integration Technique

6. For each integral below, name the technique you would use — ordinary substitution, integration by parts, a trigonometric identity, trigonometric substitution, or partial fractions — give a one-sentence reason drawn from the *form* of the integrand, and write down the first line only: the substitution, the choice of u and dv , or the shape of the decomposition. **Do not evaluate any of them.**

a) $\int \frac{x}{x^2 + 6x + 13} dx$

b) $\int x^3 \ln x dx$

c) $\int \cos^5 x dx$

d) $\int \frac{dx}{x^2 \sqrt{25 - x^2}}$

e) $\int \frac{3x - 2}{x^2 - 4x + 3} dx$

f) $\int x \sqrt{x^2 - 7} dx$

Synthesis — drawing on several topics in this set

7. Evaluate $\int_0^1 x \ln(x+1) dx$ exactly.

- a) Carry the computation through.
- b) Name the two techniques from this set that the problem required, and state the step at which each became necessary.

Want the harder problems and the worked solutions?

The *Challenge Problems* set and the *Answer Keys* for Techniques of Integration — with the reasoning written out in full — are available at tutorinmontreal.ca.