

Improper Integrals and Indeterminate Forms

Name: _____

Date: _____

Concepts

- Indeterminate Products and Differences
 - Indeterminate Powers
 - Improper Integrals over an Infinite Interval
 - Improper Integrals with an Infinite Discontinuity
 - Deciding Whether an Improper Integral Converges (theory)
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Indeterminate Products and Differences

1. Evaluate $\lim_{x \rightarrow 0^+} x^2 \ln x$.
 - a) Name the indeterminate form, and explain in one sentence why L'Hospital's rule cannot be applied to the product as it stands.
 - b) Rewrite the product as a quotient and evaluate the limit.

Indeterminate Powers

2. Evaluate each limit by taking logarithms. In each case name the indeterminate form first.

a) $\lim_{x \rightarrow \infty} \left(1 + \frac{3}{x}\right)^{2x}$

b) $\lim_{x \rightarrow 0^+} x^{3x}$

Improper Integrals over an Infinite Interval

3. Consider $\int_1^{\infty} \frac{\ln x}{x^3} dx$.

a) Write the integral as a limit of a proper definite integral.

b) Evaluate it, and say what the value means about the region lying between the curve $y = \frac{\ln x}{x^3}$ and the x -axis for $x \geq 1$.

Improper Integrals with an Infinite Discontinuity

4. A cylindrical tank of depth 4 m drains through a hole in its base. Under one model the time to empty, in seconds, is $T_1 = \int_0^4 \frac{2}{\sqrt{h}} dh$, where h is the depth of water still in the tank; a second model gives instead $T_2 = \int_0^4 \frac{2}{h} dh$.

- a) Explain why both integrals are improper, and write each as an explicit limit.
- b) Evaluate each, and say what the two results predict about the tank.

Deciding Whether an Improper Integral Converges

5. For each integral below, state which feature makes it improper, then decide whether it converges or diverges and justify your decision. A comparison with a simpler integral is enough wherever it settles the matter; you are not asked to evaluate anything a comparison already decides.

a) $\int_1^{\infty} \frac{dx}{x^2 + e^x}$

b) $\int_1^{\infty} \frac{x+3}{x^2+1} dx$

c) $\int_0^1 \frac{dx}{x + \sqrt{x}}$

Synthesis — drawing on several topics in this set

6. Consider $\int_0^1 \frac{\ln x}{\sqrt{x}} dx$.

- a) Explain why the integral is improper, and write it as an explicit limit.
- b) Evaluate $\lim_{a \rightarrow 0^+} \sqrt{a} \ln a$, naming the indeterminate form and showing the rewrite that makes L'Hospital's rule applicable.
- c) Use parts (a) and (b) to decide whether the integral converges, and give its value if it does.

Want the harder problems and the worked solutions?

The *Challenge Problems* set and the *Answer Keys* for Improper Integrals and Indeterminate Forms — with the reasoning written out in full — are available at tutorinmontreal.ca.