CEGEP Calculus II · Practice Worksheet · tutorinmontreal.ca

Applications of Integration

Name: _____ Date: _____

Concepts

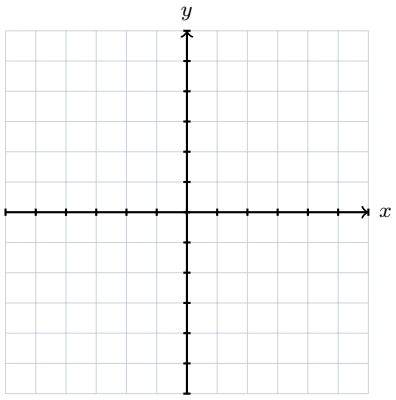
- Area Between Two Curves
- Volumes by Disks and Washers
- Volumes by Cylindrical Shells
- Choosing Between Disks, Washers and Shells (theory)
- Arc Length (theory)

Area Between Two Curves

1. The curves $y = 2x - x^2$ and $y = x^2 - 4x$ enclose a single bounded region.

a) Sketch both curves on the grid, mark the two points where they meet, and shade the enclosed region.

b) Find the area of that region.



Volumes by Disks and Washers

2. The region bounded by $y = x^2$ and $y = 2x$ is revolved about the x-axis. Sketch the region, identify the outer and inner radii of a typical washer, and find the volume of the solid generated.

Volumes by Cylindrical Shells

3. Let R be the region bounded by $y = x^2$, the x-axis, and the vertical lines $x = 1$ and $x = 3$. Using the shell method, find the volume of the solid obtained by revolving R about the y-axis. State the radius and the height of a typical shell before integrating.

Choosing Between Disks, Washers and Shells

4. Let R be the region bounded by $y = x^2 + 1$, the x-axis, and the lines $x = 0$ and $x = 2$. For each axis of revolution below, decide which method the geometry favours, write down the corresponding integral *without evaluating it*, and say in one or two sentences what makes the other method harder here. A sketch of R will settle both.

a) R revolved about the y-axis.

b) R revolved about the x-axis.

Arc Length

5. The profile of a skateboard ramp is modelled by $y = \frac{2}{3}x^{3/2}$ for $0 \le x \le 3$, with x and y measured in metres. Find the exact length of the ramp surface, then give it to three decimal places.

Synthesis — drawing on several topics in this set

6. A strip of metal is bent so that its edge follows the curve $y = \dfrac{x^3}{6} + \dfrac{1}{2x}$ for $1 \le x \le 2$, with x and y in metres. Let R be the region bounded by this curve, the x-axis, and the lines $x = 1$ and $x = 2$.

a) Show that $1 + \left(\dfrac{dy}{dx}\right)^2$ is a perfect square, and hence find the exact length of the strip.

b) Find the exact area of R.

c) Find the exact volume of the solid obtained by revolving R about the x-axis.