

Separable Differential Equations

Name: _____

Date: _____

Concepts

- Verifying a Solution of a Differential Equation
- Solving a Separable Equation
- Initial Value Problems
- Growth, Decay and Mixing Applications (theory)

Verifying a Solution of a Differential Equation

1. Consider the differential equation

$$\frac{dy}{dx} = 2x(1 + y^2).$$

- a) Show that $y = \tan(x^2 + C)$ satisfies this equation for every constant C , and state the restriction on x that the verification needs.
- b) Decide whether $y = x^3$ is a solution of the same equation. Justify your answer by substitution.

Solving a Separable Equation

2. Find the general solution of each equation. Give y explicitly in terms of x .

a) $\frac{dy}{dx} = \frac{\cos x}{y^2}$

b) $\frac{dy}{dx} = \frac{xy}{1+x^2}$

Initial Value Problems

3. Solve the initial value problem

$$\frac{dy}{dx} = 3x^2 e^{-y}, \quad y(0) = \ln 2.$$

Give y explicitly and state the largest interval on which your solution is valid.

Growth, Decay and Mixing Applications

4. A tank holds 200 L of pure water. Brine containing 3 g/L of salt runs in at 4 L/min; the tank is kept well stirred and drains at the same 4 L/min, so its volume never changes. Let $S(t)$ be the mass of salt in the tank, in grams, after t minutes.

- a) Explain why $\frac{dS}{dt} = 12 - \frac{S}{50}$, then solve for $S(t)$.
- b) How much salt is in the tank after 50 minutes? Give your answer to three decimals.
- c) Find $\lim_{t \rightarrow \infty} S(t)$ and say, with units, what it means about the liquid leaving the tank.

Synthesis — drawing on several topics in this set

5. A short-lived isotope is used to trace sediment movement in a reservoir. Its mass $A(t)$, in milligrams, decays at a rate proportional to the mass present:

$$\frac{dA}{dt} = -kA, \quad k > 0, \quad t \text{ in years.}$$

- a) Verify by substitution that $A(t) = A_0 e^{-kt}$ satisfies both the equation and the condition $A(0) = A_0$, then derive the same formula by separating the variables.
- b) The half-life of the isotope is 30 years. Find k exactly.
- c) A 12 mg sample is released. How long until 2 mg remain? Give the answer in years to three decimals, and say what it means.

Want the harder problems and the worked solutions?

The *Challenge Problems* set and the *Answer Keys* for Separable Differential Equations — with the reasoning written out in full — are available at tutorinmontreal.ca.