CEGEP Calculus II · Practice Worksheet · tutorinmontreal.ca

Sequences and Series

Name: ____ Date: __________________

Concepts

- Sequences and Their Limits
- Series, Partial Sums and the General Term
- Geometric and Telescoping Series
- The Integral Test
- The Comparison and Limit Comparison Tests
- The Ratio and Root Tests
- Alternating Series
- Absolute and Conditional Convergence (theory)
- Selecting a Convergence Test (theory)

Sequences and Their Limits

1. Decide whether each sequence converges. Where it does, find the limit; where it does not, say what the terms do instead.

a) $a_n = \dfrac{3n^2 - n}{5n^2 + 4}$

b) $b_n = \dfrac{\ln n}{\sqrt{n}}$

c) $c_n = \left(1 + \dfrac{2}{n}\right)^n$

d) $d_n = \dfrac{(-1)^n\, n}{n+1}$

Series, Partial Sums and the General Term

2. A desalination rig rinses a filter repeatedly. After n rinse cycles the total mass of salt it has removed is

$$S_n = \frac{3n}{n+2} \text{ grams}, \qquad n \geq 1.$$

a) Find the mass a_n removed by the nth cycle alone, as a formula valid for every $n \geq 1$.

b) Find $\displaystyle\sum_{n=1}^{\infty} a_n$.

c) Interpret the answer to (b) for the operator of the rig.

3. For each series, write a formula for the general term a_n valid for $n \geq 1$, then state whether the Divergence Test (the nth term test) settles the series — and if it does, give the conclusion.

a) $\frac{2}{5} - \frac{4}{7} + \frac{6}{9} - \frac{8}{11} + \cdots$

b) $\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \cdots$

c) $\frac{3}{1} + \frac{9}{4} + \frac{27}{9} + \frac{81}{16} + \cdots$

Geometric and Telescoping Series

4. Find the exact sum of each convergent series, naming the type you are using.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{2^{n+1}}{3^{n-1}}$

b) $\displaystyle\sum_{n=1}^{\infty} \frac{2}{4n^2-1}$

The Integral Test

5. Apply the Integral Test to each series. State and check the hypotheses on the companion function before integrating, and evaluate the improper integral.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{(n^2+3)^2}$

b) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\ln n}$

The Comparison and Limit Comparison Tests

6. Settle each series. Name which of the two tests you use, state the comparison series and why it converges or diverges, and show the inequality or the limit that justifies the conclusion.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{5}{2^n + n}$

b) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n} + 1}{n^2 - n + 3}$

c) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 + 2}{n^3 + 5n}$

The Ratio and Root Tests

7. Apply the Ratio Test or the Root Test — whichever the shape of the general term invites — and state the limit L and the conclusion.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{4^n}{n!}$

b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{3n+2}{2n-1}\right)^n$

c) $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2\, 5^n}$

Alternating Series

8.

a) Show that $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\, n}{n^2+4}$ converges by the Alternating Series Test. Check both hypotheses, and comment on what happens between $n = 1$ and $n = 2$.

b) The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$ converges. How many terms of it guarantee an approximation of the sum with error less than 0.001? Justify with the alternating series remainder estimate.

Absolute and Conditional Convergence

9. Classify each series as absolutely convergent, conditionally convergent, or divergent. Justify each classification by naming the test applied to $\sum |a_n|$ and, where it is needed, the test applied to $\sum a_n$ itself.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$

b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2+1}$

c) $\displaystyle\sum_{n=1}^{\infty} \frac{n\cos(n\pi)}{n+6}$

d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n}{n!}$

Selecting a Convergence Test

10. For each series below: name the *one* test you would apply and say in a sentence why the shape of the general term invites it; name the stated alternative and say why it fails or is inconclusive; then give the conclusion. Full computations are not required, but any limit you rely on must be stated.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{3n^2+1}$ (alternative offered: the Ratio Test)

b) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$ (alternative offered: the Root Test)

c) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$ (alternative offered: Direct Comparison with $\sum \frac{1}{n}$)

d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n+1}$ (alternative offered: testing $\sum |a_n|$)

e) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+3}{n^5+n^2+1}$ (alternative offered: the Ratio Test)

f) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{n+3}\right)^{n^2}$ (alternative offered: the Ratio Test)

Synthesis — drawing on several topics in this set

11. Let $S = \sum_{n=1}^{\infty} \left(\frac{1}{n(n+2)} + \frac{5}{3^n} \right)$.

a) Show that each of the two pieces converges, naming its type, and find the exact value of S.

b) Splitting a series into two like that is legitimate here but not in general. Show that $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1} \right)$ converges while neither piece does, and state the condition under which the linearity rule $\sum(a_n + b_n) = \sum a_n + \sum b_n$ may be used.