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Power Series, Taylor and Maclaurin Series

Name: ______________________ Date: ____________

Concepts

- Power Series and the Radius of Convergence
- The Interval of Convergence
- The Maclaurin Series of a Function
- Taylor Series Centred at a Point
- Building New Series from Known Ones (theory)
- Evaluating an Integral with a Series (theory)

Power Series and the Radius of Convergence

1. A ski resort models the settling of fresh snow with the power series

$$\sum_{n=0}^{\infty} \frac{(x+4)^n}{5^n\sqrt{n+1}}.$$

State the centre of the series, then apply the Ratio Test to determine its radius of convergence. Name the test and show the limit you evaluate.

The Interval of Convergence

2. Determine the interval of convergence of

$$\sum_{n=1}^{\infty} \frac{(x-3)^n}{n\,4^n}.$$

Give the centre and the radius, then test each endpoint separately, naming the test you use at each one.

The Maclaurin Series of a Function

3. Let $f(x) = \ln(1+2x)$. Working from the definition $\displaystyle\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}x^n$ — that is, by differentiating f and evaluating at 0 — find the first four non-zero terms of the Maclaurin series of f. Then give the general term, and state the radius of convergence.

Taylor Series Centred at a Point

4. Find the Taylor series of $f(x) = \dfrac{1}{x}$ centred at $a = 2$, expressed with $\displaystyle\sum_{n=0}^{\infty}$ and a general term in $(x-2)^n$. State the centre and the radius of convergence.

Building New Series from Known Ones

5. The geometric series $\dfrac{1}{1-t} = \displaystyle\sum_{n=0}^{\infty} t^n$ for $|t| < 1$ is known. **Without computing any Maclaurin coefficient from repeated derivatives**, use substitution, multiplication and term-by-term differentiation to answer the following.

a) Find the Maclaurin series of $g(x) = \dfrac{x}{1+9x^2}$ and state its radius of convergence.

b) Differentiate the geometric series term by term to obtain a series for $\dfrac{1}{(1-t)^2}$, and use it to evaluate $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2^{n-1}}$.

Evaluating an Integral with a Series

6. The function e^{-x^2} has no elementary antiderivative, so $\int_0^{1/2} e^{-x^2}\,dx$ cannot be found by the Fundamental Theorem alone.

a) Starting from the Maclaurin series of e^t, write the integral as an infinite series with a general term.

b) The series alternates. Use the Alternating Series Estimation Theorem to find how many terms guarantee an error smaller than 10^{-3}, and give the value of the integral to three decimal places.

Synthesis — drawing on several topics in this set

7. This question builds a well-known series from the geometric one and then uses it numerically.

a) Substitute into $\dfrac{1}{1-t} = \displaystyle\sum_{n=0}^{\infty} t^n$ to obtain the Maclaurin series of $\dfrac{1}{1+x^2}$, and state its radius.

b) Integrate term by term to obtain the Maclaurin series of $\arctan x$, justifying the value of the constant of integration.

c) Find the interval of convergence of the series in (b), testing both endpoints separately.

d) Use the series to approximate $\arctan \frac{1}{2}$ to three decimal places, and say how you know the error is small enough.

Want the harder problems and the worked solutions?

The *Challenge Problems* set and the *Answer Keys* for Power Series, Taylor and Maclaurin Series — with the reasoning written out in full — are available at **tutorinmontreal.ca**.