CEGEP Linear Algebra · Practice Worksheet · tutorinmontreal.ca

Vectors in the Plane and in Space

Name: ______________________________ Date: ______________

Concepts

- Geometric and Algebraic Vectors
- Norm, Direction and Direction Angles
- Vector Operations and Linear Combinations
- Representing Vectors in the Plane and in Space (theory)
- Linear Independence and Dependence
- Bases and Components (theory)
- Unit Vectors and Normalization (theory)

Geometric and Algebraic Vectors

1. Points are given in the plane and in space.

a) In the plane, $A(-2,5)$, $B(4,1)$ and $C(0,-3)$. Write $\overrightarrow{AB}$ and $\overrightarrow{BC}$ in component form.

b) Find the point D for which $\overrightarrow{CD} = \overrightarrow{AB}$.

c) The segments AB and CD are different segments of the plane. Explain, in one sentence, in what sense $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are nevertheless the same vector.

d) In space, $P(1,-2,4)$ and $Q(-3,0,5)$. Write $\overrightarrow{PQ}$ and $\overrightarrow{QP}$ in component form, and state the relation between them.

Norm, Direction and Direction Angles

2. Give every angle in degrees, to three decimals.

a) For $\vec{u} = (-5, 12)$, find $\|\vec{u}\|$ and the direction angle $\theta \in [0°, 360°)$ that $\vec{u}$ makes with the positive x-axis.

b) For $\vec{v} = (2, -3, 6)$, find $\|\vec{v}\|$, the three direction cosines, and the direction angles α, β, γ.

c) Verify that your three direction cosines satisfy $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$, and say why that had to happen.

Vector Operations and Linear Combinations

3. Let $\vec{u} = (3, -1, 2)$, $\vec{v} = (-2, 4, 0)$ and $\vec{w} = (1, 1, -5)$.

a) Compute $2\vec{u} - 3\vec{v} + \vec{w}$.

b) Compute $\|\vec{u} + \vec{v}\|$, leaving the answer in exact form.

c) Solve $3\vec{x} - \vec{u} = 2(\vec{x} + \vec{v})$ for the vector $\vec{x}$.

Representing Vectors in the Plane and in Space

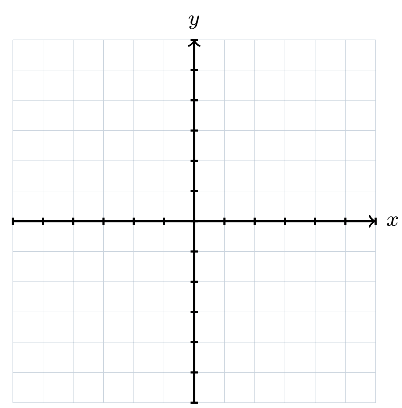
4. Use the grid below, with the origin at its centre and one square as one unit.

a) Draw $\vec{u} = (4, 1)$ and $\vec{v} = (-2, 3)$ as arrows starting at the origin.

b) Draw $\vec{u} + \vec{v}$ starting at the origin, and outline the parallelogram whose sides at the origin are $\vec{u}$ and $\vec{v}$.

c) Draw the arrow that starts at the tip of $\vec{v}$ and ends at the tip of $\vec{u}$. State its components and say which vector it represents.

d) A classmate says the arrow from the tip of $\vec{u}$ to the tip of $\vec{v}$ represents $\vec{u} - \vec{v}$ just as well. Is that right? Justify in one sentence.



Linear Independence and Dependence

5. For each set, decide whether it is linearly independent by determining all solutions of $c_1\vec{u}_1 + c_2\vec{u}_2 + \cdots = \vec{0}$. Where a set is dependent, write down an explicit dependence relation.

a) $\vec{u} = (1, 2, -1)$, $\vec{v} = (3, 1, 4)$.

b) $\vec{u} = (1, 0, 2)$, $\vec{v} = (2, -1, 1)$, $\vec{w} = (4, -1, 5)$.

c) $\vec{u} = (1, 1, 0)$, $\vec{v} = (0, 1, 1)$, $\vec{w} = (1, 0, 1)$.

Bases and Components

6.

a) Show that $\mathcal{B} = \{\vec{b}_1, \vec{b}_2\}$, with $\vec{b}_1 = (2, 1)$ and $\vec{b}_2 = (-1, 3)$, is a basis of $\mathbb{R}^2$, then find the components of $\vec{x} = (7, 7)$ in $\mathcal{B}$.

b) Show that $\mathcal{C} = \{\vec{c}_1, \vec{c}_2, \vec{c}_3\}$, with $\vec{c}_1 = (1, 0, 1)$, $\vec{c}_2 = (0, 1, 1)$ and $\vec{c}_3 = (1, 1, 0)$, is a basis of $\mathbb{R}^3$, then find the components of $\vec{y} = (4, 2, 6)$ in $\mathcal{C}$.

c) Explain why a vector cannot have two different sets of components in the same basis.

Unit Vectors and Normalization

7.

a) Find the unit vector in the direction of $\vec{u} = (6, -3, 2)$.

b) Find the vector of magnitude 21 pointing in the sense opposite to $\vec{u}$.

c) A vector $\vec{v}$ in the plane has magnitude 20 and direction angle $150°$. Write $\vec{v}$ in exact component form.

d) Explain why $\dfrac{\vec{u}}{\|\vec{u}\|}$ is undefined when $\vec{u} = \vec{0}$, and what that says about the direction of the zero vector.

Synthesis — drawing on several topics in this set

8. A courier drone flies in a straight line from a depot at $D(2,-1,0)$ to a rooftop at $R(8,5,-3)$. Coordinates are in hundreds of metres.

a) Write the displacement $\vec{d} = \overrightarrow{DR}$ in component form and find $\|\vec{d}\|$. How many metres does the drone travel?

b) Find the unit vector $\hat{d}$ in the direction of the flight, and the direction angles of $\vec{d}$ to three decimals of a degree.

c) The drone must report its position when it is 600 m from the depot along the straight route. Find the coordinates of that point.

d) Two fixed corridor directions are $\vec{p} = (4,2,-1)$ and $\vec{q} = (2,4,-2)$. Express $\vec{d}$ as a linear combination of $\vec{p}$ and $\vec{q}$, then state whether $\{\vec{p},\vec{q},\vec{d}\}$ is linearly independent, with a reason.

Want the harder problems and the worked solutions?

The *Challenge Problems* set and the *Answer Keys* for Vectors in the Plane and in Space — with the reasoning written out in full — are available at **tutorinmontreal.ca**.