CEGEP Linear Algebra · Practice Worksheet · tutorinmontreal.ca

Products of Vectors

Name: ______________________________ Date: ______________

Concepts

- The Dot Product and the Angle Between Vectors
- Orthogonal Projection
- The Cross Product
- The Scalar Triple Product
- Areas and Volumes from Vector Products (theory)
- Choosing the Right Product (theory)

The Dot Product and the Angle Between Vectors

1. A drone's three guy wires leave the same anchor point along the directions

$$\vec{u} = (3, 4, 0), \qquad \vec{v} = (0, -4, 3), \qquad \vec{w} = (4, -3, 0),$$

measured in metres.

a) Compute $\vec{u} \cdot \vec{v}$ and $\vec{u} \cdot \vec{w}$.

b) Find the angle between $\vec{u}$ and $\vec{v}$, to the nearest tenth of a degree.

c) State what the value of $\vec{u} \cdot \vec{w}$ tells you about $\vec{u}$ and $\vec{w}$, and say how you could have predicted the sign of $\vec{u} \cdot \vec{v}$ from your answer to (b).

Orthogonal Projection

2. Let $\vec{u} = (1, 5, -1)$ and $\vec{v} = (2, 1, -2)$.

a) Find the scalar component of $\vec{u}$ along $\vec{v}$, that is $\dfrac{\vec{u} \cdot \vec{v}}{\|\vec{v}\|}$.

b) Find the orthogonal projection $\operatorname{proj}_{\vec{v}} \vec{u}$.

c) Write $\vec{u}$ as the sum of a vector parallel to $\vec{v}$ and a vector orthogonal to $\vec{v}$, and verify the orthogonality.

The Cross Product

3. Let $\vec{u} = (2, -1, 3)$ and $\vec{v} = (1, 4, -2)$.

a) Compute $\vec{u} \times \vec{v}$ using the 3×3 determinant.

b) Verify that your answer is orthogonal to both $\vec{u}$ and $\vec{v}$.

c) Write down $\vec{v} \times \vec{u}$ without computing a second determinant, and name the property you used.

The Scalar Triple Product

4. Let $\vec{u} = (1, 2, -1)$, $\vec{v} = (3, 0, 2)$ and $\vec{w} = (2, -1, 1)$.

a) Evaluate $\vec{u} \cdot (\vec{v} \times \vec{w})$ as a single 3×3 determinant.

b) Confirm the value by computing $\vec{v} \times \vec{w}$ first and then taking its dot product with $\vec{u}$.

c) State what your value tells you about the three vectors.

Areas and Volumes from Vector Products

5. A sheet-metal shop marks the points $A(1, 0, 2)$, $B(3, 2, 1)$, $C(2, -1, 4)$ and $D(2, 1, 5)$, coordinates in metres.

a) Find the area of the parallelogram determined by $\vec{AB}$ and $\vec{AC}$.

b) Find the area of the triangle ABC.

c) Find the volume of the parallelepiped whose edges from A are $\vec{AB}$, $\vec{AC}$ and $\vec{AD}$.

Choosing the Right Product

6. For each geometric question below, state which *one* of the dot product, the cross product and the scalar triple product settles it, and give one sentence saying why each of the other two cannot. Compute nothing.

a) Two struts leave a joint along $\vec{a}$ and $\vec{b}$. Is the angle between them acute or obtuse?

b) Three cables leave an anchor along $\vec{a}$, $\vec{b}$ and $\vec{c}$. Do all three lie in one plane?

c) A camera arm must point at right angles to both $\vec{a}$ and $\vec{b}$. Which direction is that?

d) A triangular solar panel has edge vectors $\vec{a}$ and $\vec{b}$ from one corner. How much surface does it present?

Synthesis — drawing on several topics in this set

7. Let $\vec{u} = (1, -2, 2)$ and $\vec{v} = (4, 0, -3)$.

a) Find the angle between $\vec{u}$ and $\vec{v}$, to the nearest tenth of a degree.

b) Find $\operatorname{proj}_{\vec{v}} \vec{u}$.

c) Find $\vec{u} \times \vec{v}$ and the area of the parallelogram it determines.

d) Verify for these vectors that $\|\vec{u} \times \vec{v}\|^2 = \|\vec{u}\|^2\|\vec{v}\|^2 - (\vec{u} \cdot \vec{v})^2$, and say which of (a)–(c) this identity connects.