CEGEP Linear Algebra · Practice Worksheet · tutorinmontreal.ca

Lines and Planes in Space

Name: ______________________________ Date: ______________

Concepts

- Vector, Parametric and Symmetric Equations of a Line
- Vector, Parametric and Cartesian Equations of a Plane
- The Relative Position of Two Lines
- The Relative Position of Lines and Planes
- Intersections of Lines and Planes
- Distances Between Points, Lines and Planes
- Angles Between Lines and Planes (theory)

Vector, Parametric and Symmetric Equations of a Line

1. A surveyor sights along a straight guy wire that passes through the points $A(2,-1,3)$ and $B(5,1,-1)$, coordinates in metres.

a) Give a direction vector $\vec{d}$ for the line AB, then write a vector equation of the line.

b) Write parametric equations of the line.

c) Write symmetric equations of the line.

d) Decide, showing your reasoning, whether $P(11,5,-9)$ and $R(8,3,-6)$ lie on the line.

Vector, Parametric and Cartesian Equations of a Plane

2. Three anchor bolts of a solar panel sit at $A(1,0,2)$, $B(3,1,2)$ and $C(2,-1,4)$. The panel lies in the plane π through these three points.

a) Write a vector equation of π.

b) Write parametric equations of π.

c) Find a normal vector to π and hence its cartesian equation in the form $ax+by+cz+d=0$.

d) Decide whether $E(4,2,1)$ and $F(0,1,0)$ lie in π.

The Relative Position of Two Lines

3. Determine the relative position of each pair of lines. Where they meet, give the point of intersection; where they do not, say why not.

a) $\ell_1 : (x, y, z) = (1, 0, 2) + t(2, 1, -1)$ and $\ell_2 : (x, y, z) = (5, 3, -1) + s(-4, -2, 2)$.

b) $\ell_3 : (x, y, z) = (2, -1, 3) + t(1, 2, -1)$ and $\ell_4 : (x, y, z) = (1, 2, 1) + s(2, -1, 1)$.

The Relative Position of Lines and Planes

4. Let $\pi :\ 2x - y + 3z - 4 = 0$. For each line below, determine its position relative to π — contained in π, parallel to π and disjoint from it, or meeting π in exactly one point. State the test you used, and give the point of intersection where there is one.

a) $\ell_1 : (x, y, z) = (1, 0, 1) + t(1, 2, 0)$

b) $\ell_2 : (x, y, z) = (2, 3, 1) + t(1, 2, 0)$

c) $\ell_3 : (x, y, z) = (0, 2, 0) + t(1, -1, 1)$

Intersections of Lines and Planes

5.

a) A laser beam travels along $\ell : (x, y, z) = (4, -2, 1) + t(-1, 3, 1)$ and strikes the screen lying in the plane $\pi : \ x + y + z - 6 = 0$. Find the point where it meets the screen.

b) Three flat panels meet along the edges of a corner bracket:

$$\pi_1 : \ x + y + z - 6 = 0, \qquad \pi_2 : \ 2x - y + z - 3 = 0, \qquad \pi_3 : \ x + 2y - z - 2 = 0.$$

Using row reduction on the augmented matrix, find all points common to the three planes, and say what the type of solution set means geometrically.

Distances Between Points, Lines and Planes

6. Compute each of the following exactly; all coordinates are in metres.

a) The distance from the sensor at $P(3,-2,1)$ to the wall $\pi:\ 2x-y+2z+5=0$.

b) The distance from the point $Q(2,2,4)$ to the line $\ell:(x,y,z)=(1,0,2)+t(2,1,-2)$.

c) The distance between the parallel planes $\pi_1:\ x-2y+2z-3=0$ and $\pi_2:\ x-2y+2z+6=0$.

Angles Between Lines and Planes

7. Give each angle to three decimal places where it is not exact, and take every angle to be the acute (or right) one.

a) Find the angle between the lines of directions $\vec{d_1} = (1, -2, 2)$ and $\vec{d_2} = (2, 2, -1)$. Note that $\vec{d_1} \cdot \vec{d_2} < 0$ and explain what you do about that.

b) Find the angle between the line of direction $\vec{u} = (2, -1, 2)$ and the plane $\pi :\ x + 2y + 2z - 7 = 0$.

c) Find the angle between the planes $\pi :\ x + 2y + 2z - 7 = 0$ and $\sigma :\ 2x - 2y + z + 3 = 0$.

d) The formula in (a) and (c) uses a cosine, the one in (b) a sine. Explain why, in one or two sentences.

Synthesis — drawing on several topics in this set

8. A triangular sail is stretched between the mast fittings $A(1,0,0)$, $B(0,2,0)$ and $C(0,0,3)$, coordinates in metres; the sail lies in the plane π through those three points. A guy rope runs along $\ell : (x,y,z) = (0,0,4) + t(1,-1,-2)$.

a) Find the cartesian equation of π in the form $ax + by + cz + d = 0$.

b) Find the point where the rope's line meets the plane of the sail.

c) Find the acute angle between the rope and the sail, to three decimal places.

d) Find the distance from the mast foot at the origin to the plane of the sail.