

Angles, Lines and Their Relationships

Name: _____

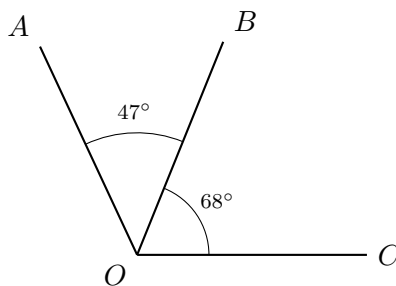
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Concepts

- Angles
- Classifying Angles
- Constructing Perpendicular and Parallel lines (theory)
- Constructing a Bisector (theory)
- Constructing a Median (theory)
- Constructing a Perpendicular Bisector (theory)
- Geometry (theory)
- Relationships Between Angles
- Relationships Between Two Lines
- Special Lines (theory)
- Types of Lines

Angles

1. Three light beams leave the same point O on a stage rig, along the rays OA , OB and OC shown below.



- Name the angle formed by the rays OA and OC using three letters, and give its vertex and its two sides.
- Find $m\angle AOC$.
- The beam OB is now turned, staying between OA and OC , until $m\angle AOB = 60^\circ$; the beams OA and OC do not move. What is $m\angle BOC$ then?

Classifying Angles**2.**

- a) Classify each of these angles as acute, right, obtuse, straight, reflex or a full angle: 89° , 90° , 151° , 180° , 237° , 360° .
- b) At exactly 4 o'clock, the two hands of a clock form an angle. Give its measure and classify it.

Constructing Perpendicular and Parallel lines

3. Antoine has a straight line d drawn on his page and a point P that is not on d . He wants to construct the line through P that is parallel to d , using only a ruler and a set square. His four steps are listed here in the wrong order:

- a) Slide the set square along the ruler, keeping it pressed against the ruler, until the edge that was lying on d passes through P .
- b) Hold the ruler firmly against a second edge of the set square, and do not move the ruler again.
- c) Trace the line along the edge that now passes through P .
- d) Place one edge of the set square exactly along the line d .

Give the correct order of the four steps, then explain in one or two sentences why the line obtained really is parallel to d .

Constructing a Bisector

4. In a stained-glass panel, the ray BD is the bisector of $\angle ABC$.
- a) If $m\angle ABD = 38^\circ$, find $m\angle ABC$.
 - b) In a second panel Rosalie measures $m\angle ABD = 38^\circ$ and $m\angle DBC = 39^\circ$, and says that BD is the bisector there as well. Is she right? Explain.
 - c) In the compass-and-ruler construction of a bisector, the first two arcs are drawn from the vertex and must have the *same* radius. Say what would go wrong if the two radii were different.

Constructing a Median

5. In $\triangle KLM$, the median drawn from the vertex K meets the side LM at the point N , and LM measures 18 cm.
- a) Find the length of $\overline{LN}$.
 - b) Charlotte instead draws the segment from K that meets LM at a right angle, and calls it the median. Name the segment she actually drew, and say what would have to be true about the triangle for her segment and the median to be the same one.

Constructing a Perpendicular Bisector

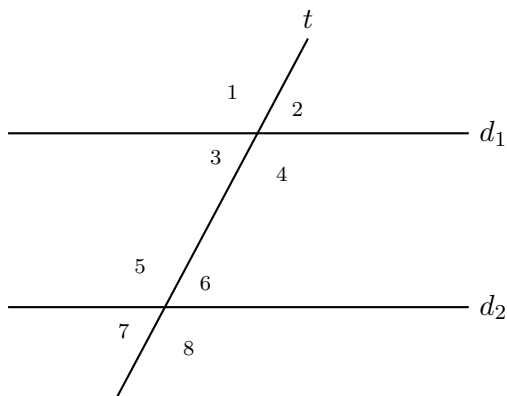
6. Two goal posts P and Q are planted 14 m apart on a level field, and a coach constructs the perpendicular bisector of the segment $\overline{PQ}$.
- a) At what distance from P does the perpendicular bisector cross $\overline{PQ}$?
 - b) What angle does it make with $\overline{PQ}$, and how is that angle classified?
 - c) A cone is placed at a point T on the perpendicular bisector, 9 m away from P . How far is the cone from Q ? Justify your answer.

Geometry

7. Recognizing *which* relationship applies is most of the work in geometry. For each situation below, name the object or relationship you would use, and state the one thing that must be checked before you are allowed to use it.
- a) Two angles sit side by side and their outer sides together form one straight line.
 - b) Two straight lines cross, and you are looking at the two angles that face each other across the crossing point.
 - c) Two lines are cut by a third line, and you want to conclude that a certain pair of angles is congruent.
 - d) A segment is drawn from a vertex of a triangle to a point on the opposite side, and you want to conclude that it cuts that side into two equal parts.

Relationships Between Angles

8. In the figure below, $d_1 \parallel d_2$ and the line t cuts them both. The eight angles formed are numbered.



Given that $m\angle 3 = 62^\circ$, find the measure of $\angle 1$, of $\angle 6$ and of $\angle 8$. For each one, name the relationship you used.

Relationships Between Two Lines

9. A street plan shows several straight streets. For each pair below, name the relationship between the two lines (distinct parallel, coincident, intersecting, or perpendicular).

- a) Rue des Ormes and Rue Bellevue never meet, however far each one is extended.
- b) Boulevard Central crosses Rue des Ormes, and the four angles at the crossing are all right angles.
- c) Rue du Parc and Avenue Sud cross at a single point, forming angles of 55° and 125° .
- d) On an older map the same road is drawn twice, once as Rue Principale and once as Route 138, one exactly on top of the other over its whole length.

Then say which of these pairs are also intersecting lines, and explain the one that surprises people.

Special Lines

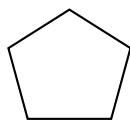
10. Four special lines are described inside $\triangle ABC$. Name each one — median, height, angle bisector, or perpendicular bisector.

- a) It starts at A , ends on $\overline{BC}$, and meets $\overline{BC}$ at a right angle.
- b) It starts at A and ends at the midpoint of $\overline{BC}$.
- c) It starts at A and cuts $\angle BAC$ into two congruent angles.
- d) It crosses $\overline{BC}$ at its midpoint and is perpendicular to $\overline{BC}$, but it is not required to pass through any vertex.

Then say which of the four is the only one that does not have to pass through a vertex of the triangle, and why.

Types of Lines

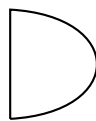
11. Four lines are drawn below.



(a)



(b)



(c)

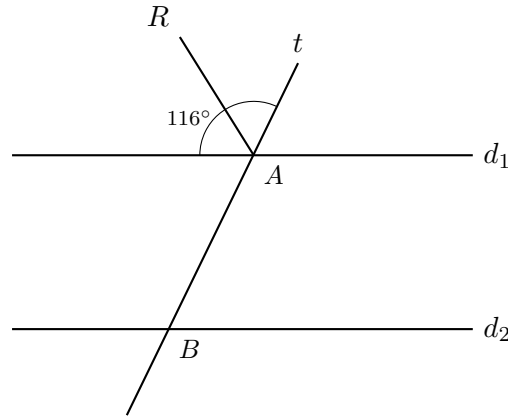


(d)

- a) For each one, say whether it is open or closed, and whether it is a broken line, a curved line, or made of both straight and curved parts.
- b) Which of the four is a polygon?
- c) A straight line, a ray and a line segment are drawn on the same page. Which of the three has a length that can be measured, and why?

Synthesis — drawing on several sheets in this topic

12. In the figure, $d_1 \parallel d_2$ and the line t cuts them at A and at B . The angle at A marked by the arc measures 116° in total — the arc runs all the way from t round to d_1 — and the ray AR is its bisector.



- a) Find the measure of the angle between the ray AR and the part of t that goes upwards from A , and classify that angle.
- b) Find the measure of the angle at B that lies above d_2 and to the left of t , and name the relationship you used.
- c) Is AR parallel to d_2 ? Justify your answer using (a) and (b).

Want the harder problems and the worked solutions?

The *Challenge Problems* set and the *Answer Keys* for Angles, Lines and Their Relationships — with the reasoning written out in full — are available at tutorinmontreal.ca.