Secondary 1 Mathematics · Practice Worksheet · tutorinmontreal.ca

Polygons and Geometric Constructions

Name: _____ Date: _____

Concepts

- Classifying Polygons
- Congruent and Similar Figures
- Constructing Polygons
- Constructing Regular Polygons
- Constructing a Rhombus and a Parallelogram
- Constructing a Square and a Rectangle
- Constructing a Trapezoid
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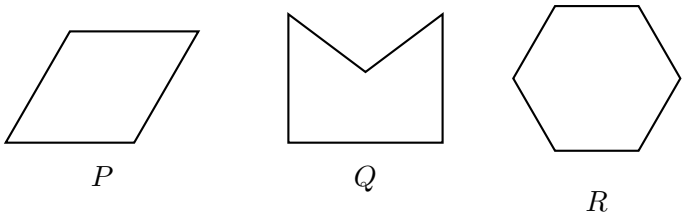
- Polygons (theory)
- Regular Polygons
- Similarity, Congruence, and Equivalence (theory)
- The Construction of a Height (theory)

Classifying Polygons

1. A graphic designer is preparing three signs for a new skatepark. Each sign has the shape drawn below. On the plan, the four sides of P are congruent and the six sides of R are congruent.



P Q R

a) Name each of the three polygons according to its number of sides.

b) One of them is non-convex. Say which, and name the feature that makes it non-convex.

c) Exactly one of the three is a regular polygon. Say which one, and for each of the other two give a condition of a regular polygon that it fails.

Congruent and Similar Figures

2. A bubble-tea shop prints its loyalty card as a rectangle 9 cm long and 5 cm wide. For the window display, the shop orders a poster of the same card in which every length is multiplied by 4. It also prints a small sticker measuring 5 cm by 9 cm.

a) Give the length and the width of the poster.

b) Is the poster congruent to the card, similar to it, or neither? Justify your answer by checking the angles and the sides.

c) Is the sticker congruent to the card? Explain.

Constructing Polygons

3. Using a ruler, a compass and a protractor, construct the quadrilateral $ABCD$ below, at true size, in the space provided.

- $AB = 6$ cm;
- $m\angle DAB = 70°$ with $AD = 4$ cm;
- $m\angle ABC = 110°$ with $BC = 5$ cm;
- then join C to D.

a) Measure DC, $m\angle ADC$ and $m\angle BCD$ on your drawing.

b) Add the four angle measures. What total should you get, and does your drawing agree?

c) Look at the sides AD and BC. What do you notice, and what does it tell you about the kind of quadrilateral you have built?

Constructing Regular Polygons

4. A community garden is laid out as a regular hexagon inscribed in a circle. Construct a scale drawing with a compass and a ruler:

- draw a circle of radius 3.5 cm and mark its centre O;
- without changing the compass opening, place the point anywhere on the circle and step that opening around the circle, marking each new point;
- join the six points in order.

a) Measure a side of your hexagon and one of its interior angles.

b) Join O to two neighbouring vertices and measure the angle formed at O — the central angle.

c) Explain why the compass opening fits exactly six times around the circle and never five or seven.

Constructing a Rhombus and a Parallelogram

5. Construct the rhombus $ABCD$ with a side of 4 cm and $m\angle DAB = 60°$, in the space below. Draw $AB = 4$ cm, mark $60°$ at A and place D at 4 cm along that ray, then use the compass opened to 4 cm from B and from D to locate C.

a) Draw the two diagonals AC and BD and measure them.

b) Verify with a protractor and a ruler that the diagonals cut each other at right angles and that each one cuts the other in half.

c) The diagonal BD separates the rhombus into two triangles. Name them and explain why each one is equilateral.

Constructing a Square and a Rectangle

6. Construct a square $ABCD$ with a side of 5 cm using only a ruler and a compass — no protractor. Draw $AB = 5$ cm on a line, then construct the perpendicular to AB at A like this: with centre A and any opening, draw arcs cutting the line on both sides of A, at X and Y; then with a larger opening draw arcs centred at X and at Y that meet at Z; the line AZ is perpendicular to AB. Place D at 5 cm from A on AZ, then locate C with two arcs of radius 5 cm, one centred at B and one centred at D; those two arcs cross at A and at one other point, and that other point is C.

a) Measure the two diagonals of your square. What do you notice?

b) Check where the diagonals cross and at what angle.

c) Which of these properties would still hold if you had built a 7 cm by 5 cm rectangle instead?

Constructing a Trapezoid

7. Construct the trapezoid $ABCD$ below, at true size, with a ruler and a set square (or a compass).

- $AB = 8$ cm is the longer of the two parallel sides;
- $DC = 5$ cm is parallel to AB and 4 cm above it;
- the side AD is perpendicular to AB, so D sits directly above A.

a) State the measures of $m\angle DAB$ and $m\angle ADC$ without measuring them, and say how you know.

b) Measure $m\angle ABC$ and $m\angle BCD$ on your drawing, then check that the four angles total $360°$.

c) Which pair of sides is parallel, and how can you check it on your drawing with a set square?

Constructing a Triangle

8. Construct $\triangle ABC$ with $AB = 7$ cm, $BC = 5$ cm and $AC = 6$ cm, using a ruler and a compass only.

a) Write down, in order, the compass steps you used.

b) Measure the three angles of your triangle and check that they total $180°$.

c) Anh builds a triangle from the same three lengths but starts from the 6 cm side. Will her triangle be congruent to yours? Explain.

Plane figures

9. Sort the six figures below into three groups: (i) plane figures whose boundary is made only of line segments; (ii) plane figures whose boundary includes at least one curved part; (iii) drawings that are not closed figures at all. Justify each choice in one line.

a) A circle of radius 3 cm.

b) A pentagon.

c) A path of four segments joined end to end whose last endpoint does not return to the start.

d) A rectangle with a half-disc drawn on top of one side, forming a single closed outline.

e) A single segment of length 8 cm.

f) A right triangle.

Polygons

10. A polygon is a closed plane figure formed by line segments joined end to end, where the segments meet only at their endpoints. Four figures are described below. Say which ones are polygons, and for each one that is not, name the condition of the definition that it fails.

a) Five segments joined end to end, the last returning to the starting point, none of them crossing.

b) A closed figure made of three segments and one arc.

c) Six segments joined end to end and closed, but two of them cross each other partway along.

d) Four segments joined end to end, the fourth ending 2 cm away from the starting point.

Regular Polygons

11. A stop-motion studio builds a set piece in the shape of a regular octagon with a side of 4.5 cm.

a) State the two conditions that make this polygon regular, and give the total length of its outline.

b) Draw a regular octagon roughly and join one chosen vertex to every other vertex it is not already joined to. Into how many triangles is the octagon cut?

c) Use your answer to b) to find the sum of the eight interior angles, then the measure of one interior angle.

Similarity, Congruence, and Equivalence

12. Two figures are **congruent** when they have the same shape and the same size, **similar** when they have the same shape, and **equivalent** when they have the same area. For each pair below, say which of the three words apply — more than one may apply — and justify briefly.

a) Two squares, each with a side of 6 cm.

b) A rectangle 4 cm by 9 cm and a square with a side of 6 cm.

c) A rectangle 3 cm by 5 cm and a rectangle 6 cm by 10 cm.

d) Two triangles cut from the same sheet of cardboard, one being the mirror image of the other.

The Construction of a Height

13. A **height** of a triangle drawn from a vertex is the perpendicular segment from that vertex to the line containing the opposite side.

Construct $\triangle DEF$ with $DE = 8$ cm, $m\angle EDF = 30°$ and $m\angle DEF = 20°$. Then construct the height from D to the side EF.

a) Give $m\angle DFE$ and say what kind of triangle this is.

b) Describe what you had to do to the side EF before the foot of the height from D could be marked, and explain why.

c) Of the three heights of this triangle, which one has its foot inside the triangle? Justify.

Synthesis — drawing on several sheets in this topic

14. Draw a circle with centre O and radius 4 cm, and construct the regular hexagon $ABCDEF$ inscribed in it by stepping the radius around the circle. Then join A to C, C to E and E to A.

a) Explain why $\triangle ACE$ has three congruent sides, using the triangles $\triangle ABC$, $\triangle CDE$ and $\triangle EFA$ that you have just cut off.

b) Is $\triangle ACE$ a regular polygon? Is it similar to the hexagon? Justify each answer.

c) Give the central angle of the hexagon and the central angle of $\triangle ACE$, and check that each one divides $360°$ exactly.