

Geometric Constructions

Name: _____

Date: _____

Concepts

- Constructing Perpendicular and Parallel lines (theory)
- Constructing Polygons
- Constructing Regular Polygons
- Constructing a Bisector (theory)
- Constructing a Circle
- Constructing a Median (theory)
- Constructing a Perpendicular Bisector (theory)
- Constructing a Square and a Rectangle
- Constructing a Trapezoid
- Constructing a Triangle
- The Construction of a Height (theory)

Constructing Perpendicular and Parallel lines

1. Draw a line d right across your page, and mark a point P about 4 cm above it. Use only a compass and an unmarked straightedge — no protractor, no ruler markings.

- a) Construct the line through P that is perpendicular to d . Call it p .
- b) Construct the line through P that is parallel to d .
- c) Explain what guarantees that the line you drew in b) never meets d .

Constructing Polygons

2. Using a ruler and a protractor, construct the quadrilateral $MATH$ with

$$MA = 6 \text{ cm}, \quad \angle A = 110^\circ, \quad AT = 4.5 \text{ cm}, \quad \angle T = 75^\circ, \quad TH = 5 \text{ cm}.$$

Build it in that order, then join H back to M .

- a) Measure $\angle M$ and $\angle H$ on your drawing.
- b) Without measuring anything else, say what $\angle M + \angle H$ must be, and use it to check your two measurements.

Constructing Regular Polygons

3. Open your compass to 4 cm and draw a circle. Without changing the opening, put the compass point anywhere on the circle and mark an arc on the circle; move the point to that new mark and repeat, all the way round.

- a) Join the marks in order. Which regular polygon have you built? Measure a side to check.
- b) Draw the segments from the centre to each vertex. What kind of triangle is each of the six pieces, and why?
- c) Use b) to explain why the marks land exactly back on the starting point, instead of overshooting or falling short.

Constructing a Bisector

4. Draw an angle $\angle BAC$ of 76° with your protractor, making both arms about 7 cm long.
- a) Construct its bisector with a compass and straightedge only.
 - b) Measure the two halves with your protractor. What should each one read?
 - c) Mark a point P on the bisector, about 3 cm from A . Measure the shortest distance from P to each arm. What do you notice, and why does the construction force it?

Constructing a Circle

5. Construct a circle with centre O and diameter 7 cm.

- a) What compass opening did you use, and why?
- b) Draw a chord AB of length 6 cm in your circle (not through the centre).
- c) Construct the perpendicular from O to the chord AB , and call its foot M . Measure AM and MB . State the property your measurements suggest.

Constructing a Median

6. Construct a triangle ABC with $AB = 9$ cm, $BC = 7$ cm and $AC = 6$ cm.

- a) Construct the midpoint of each side, and draw the three medians.
- b) What do you notice about the three medians? Call the point G .
- c) Measure AG and the distance from G to the midpoint of BC . What relationship do you see between the two?

Constructing a Perpendicular Bisector

7. Draw a segment AB of length 8 cm.

- a) Construct its perpendicular bisector with a compass and straightedge.
- b) Mark two points P and Q on that line, one on each side of AB . Measure PA , PB , QA and QB , then state the property these four measurements show.
- c) On a map, two skateparks stand 8 cm apart. The city wants a water fountain the same distance from both. Where can it be placed? How many possible positions are there?

Constructing a Square and a Rectangle

8. Construct a rectangle $ABCD$ with $AB = 6$ cm and $BC = 4$ cm, using a compass and straightedge for the right angles rather than the corner of your ruler.

- a) Describe the order of your steps.
- b) Draw both diagonals and measure them. What do you notice?
- c) Mark the point where the diagonals cross and measure the four pieces. State the two properties of a rectangle's diagonals that your measurements support.

Constructing a Trapezoid

9. Construct a trapezoid $PQRS$ whose parallel sides measure $PQ = 7$ cm and $SR = 4$ cm, with a height of 3 cm between them. Place S so that it sits 1.5 cm to the right of P .

- a) Describe how you used a perpendicular and a parallel construction to place the shorter side.
- b) Calculate the area of your trapezoid.
- c) Amélie builds a different trapezoid with the same two parallel sides and the same height, but with S placed 3 cm to the right of P . Without measuring, say how its area compares with yours, and why.

Constructing a Triangle

10. Build each triangle accurately, then answer the question that follows it.

- a) Construct $\triangle ABC$ with $AB = 7$ cm, $BC = 6$ cm and $AC = 4$ cm, using a compass. Measure its largest angle. Which side is it opposite?
- b) Construct $\triangle DEF$ with $DE = 8$ cm, $\angle D = 40^\circ$ and $\angle E = 65^\circ$. Find $\angle F$ without measuring it, then check with your protractor.

The Construction of a Height

11. Construct $\triangle ABC$ with $BC = 6$ cm, $\angle B = 130^\circ$ and $AB = 4$ cm.

- a) Construct the height from A to the side BC . Something unusual happens — describe it, and say what you must do to the side BC before the construction will work.
- b) Measure that height and calculate the area of the triangle.
- c) Construct the height from B to the side AC . Does it behave the same way? Explain the difference in terms of the angles of the triangle.

Synthesis — drawing on several sheets in this topic

12. Construct $\triangle ABC$ with $AB = 8$ cm, $BC = 7$ cm and $AC = 5$ cm.

- a) Construct the perpendicular bisector of AB and the perpendicular bisector of BC . Call their crossing point O .
- b) Draw the circle with centre O and radius OA . What do you notice about B and C ?
- c) Explain, using the property of a perpendicular bisector, why this had to happen. What would the perpendicular bisector of the third side, AC , do if you constructed it too?

Want the harder problems and the worked solutions?

The *Challenge Problems* set and the *Answer Keys* for Geometric Constructions — with the reasoning written out in full — are available at tutorinmontreal.ca.