

The Area and Volume of Solids

Name: _____

Date: _____

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Constructing the Net of Prisms

1. A chocolate bar is packaged in a right prism whose base is a triangle with sides 5 cm, 5 cm and 8 cm. The prism is 20 cm long.
 - a) List every piece that appears in the net of this package, giving the dimensions of each rectangle.
 - b) How many faces does the package have in total?
 - c) The three rectangles are drawn side by side as a single strip. What are the dimensions of that strip?

Constructing the Nets of Pyramids

- 2.** A paper lantern is a regular pyramid with a square base of side 10 cm. Each of its four triangular faces has a slant height of 12 cm, measured from the middle of a base edge up to the apex.
- a) List the pieces of the net and give the dimensions of each one.
 - b) Find the area of paper used, including the base.
 - c) Which measurement of the pyramid appears in the net — the height of the pyramid or the slant height of a face? Explain.

Constructing the Nets of Solids

- 3.** A packaging workshop keeps one net on file for each shape it sells. For each solid below, say how many pieces its net contains and what shape each piece is.
- a) A cube.
 - b) A right prism with a regular hexagonal base.
 - c) A cylinder.
 - d) A pyramid with a triangular base.

Decomposable and Truncated Solids

4. A wooden toy train engine is made of two rectangular prisms glued together: the body measures $18\text{ cm} \times 8\text{ cm} \times 6\text{ cm}$, and the cab, which measures $8\text{ cm} \times 8\text{ cm} \times 7\text{ cm}$, sits on top of one end of the body.

- a) Find the volume of wood in the engine.
- b) The wood has a mass of 0.6 g per cubic centimetre. Find the mass of the engine.

Finding the Area of a Solid Using Algebra

5. A locker organiser is a right prism with a square base of side x centimetres and a height of 9 cm.

- a) Write a simplified expression for the total area of the prism in terms of x .
- b) Use your expression to find the total area when $x = 5\text{ cm}$.
- c) Check your answer to b) by computing the area face by face.

Prisms

6. A bird feeder is a right prism whose base is a trapezoid with parallel sides of 10 cm and 6 cm and a height of 4 cm. The prism is 15 cm long.

- a) Name the solid precisely and say how many faces, vertices and edges it has.
- b) Find the area of the base.
- c) Find the volume of seed the feeder holds when it is full.

Pyramids

7. The roof of a garden gazebo is a regular pyramid with a regular octagonal base.

- a) How many faces, vertices and edges does it have?
- b) Verify Euler's relation $F + V - E = 2$ for this solid.
- c) Which of its edges are congruent to one another, and which are not?

Solids

8. Here are five solids: a cube, a cylinder, a pyramid with a triangular base, a sphere and a right prism with a hexagonal base.

- a) Sort them into polyhedra and solids that are not polyhedra.
- b) State, in your own words, what makes a solid a polyhedron.
- c) Explain why a solid that has a curved surface can never be a polyhedron.

Solids with Curved Surfaces

9. Copy and complete the table for the three solids with curved surfaces.

Solid	Number of flat faces	Number of curved surfaces
Cylinder		
Cone		
Sphere		

- a) Fill in the six entries.
- b) Which of the three has a net containing a rectangle, and what does that rectangle become when the net is folded?

Tetrahedrons

10. A puzzle piece is a regular tetrahedron with an edge of 9 cm.

- a) How many faces, vertices and edges does it have?
- b) A tetrahedron is a pyramid. What shape is its base, and what makes it different from the other pyramids you have met?
- c) One of its faces has an area of 35 cm^2 , to the nearest square centimetre. Find the total area of the puzzle piece.

The Area and the Volume of Solids - Secondary 1 and 2

11. For each quantity below, say whether it is a length, an area or a volume, and give a sensible unit for it.

- a) The paint needed to cover the four walls of a bedroom.
- b) The strip of trim to be nailed around a window frame.
- c) The water a rectangular aquarium holds.
- d) The wrapping paper needed for a gift box.
- e) The distance all the way around the base of a cylindrical drum.

The Area of Cylinders

12. A cylindrical soup can has a radius of 5 cm and a height of 12 cm. Give each answer to the nearest tenth of a square centimetre.

- a) Find the area of the label that wraps around the can (the lateral area).
- b) Find the combined area of the two circular ends.
- c) Find the total area of metal in the can.

The Area of Decomposable Solids

13. A display podium for a science fair is made of two rectangular prisms. The lower block measures $80\text{ cm} \times 40\text{ cm} \times 20\text{ cm}$. The upper block measures $40\text{ cm} \times 40\text{ cm} \times 20\text{ cm}$ and sits squarely on one end of the lower block's top face. Every visible outside surface is to be varnished, including the underside.

- a) Find the total area of the two blocks taken separately.
- b) Explain which parts of that total are not varnished, and find the area to be varnished.

The Area of Prisms

14. A doorstep is a right prism whose base is a right triangle with legs of 5 cm and 12 cm and a hypotenuse of 13 cm. The prism is 20 cm long.

- a) Find the area of one triangular base.
- b) Find the lateral area, using the perimeter of the base.
- c) Find the total area, and also the volume of the doorstep.

The Area of Pyramids

15. The roof of a birdhouse is a regular pyramid with a square base of side 14 cm. Each triangular face has a slant height of 24 cm.

- a) Find the area of one triangular face.
- b) Find the lateral area of the roof.
- c) Find the total area of the pyramid, base included.

The Area of Truncated Solids

16. A wooden pen holder starts as a right prism with a rectangular base 6 cm by 4 cm, then its top is sawn off on a slant. Two of its vertical edges are 10 cm long and the other two are 7 cm long. The slanted top face is a rectangle measuring 4 cm by 6.7 cm (to the nearest tenth).

- a) Two of the side faces are trapezoids. Find the area of one of them.
- b) Find the areas of the other two side faces and of the base.
- c) Find the total area of the pen holder, to the nearest tenth of a square centimetre.

The Area of a Cube

17.

- a) A storage cube has an edge of 7 cm. Find its total area.
- b) A second cube has a total area of 150 cm^2 . Find the length of its edge.
- c) How much greater is the total area of the first cube than that of the second?

Synthesis — drawing on several sheets in this topic

18. A storage bin is a right prism with a rectangular base 60 cm by 40 cm and a height of 50 cm. A cylindrical channel of radius 10 cm is bored straight through it from the top face to the bottom face, so that cables can pass through. Give each answer to the nearest tenth.

- a) Find the volume of material left in the bin.
- b) Find the lateral area of the outside of the prism, and the combined area of the top and bottom faces once the two circular holes are taken out.
- c) Find the total area of the finished bin, remembering the wall of the channel.

Want the harder problems and the worked solutions?

The *Challenge Problems* set and the *Answer Keys* for The Area and Volume of Solids — with the reasoning written out in full — are available at tutorinmontreal.ca.