

Probability

Name: _____

Date: _____

Concepts

- Modes of Representation and the Enumeration of Possible Outcomes
- Probability
- Random Experiments Where Order Matters and Where Order Does Not Matter
- Random Experiments With One or More Steps
- Random Experiments With and Without Replacement
- Sample Space
- The Concept of "Or" and "And" in Probability
- The Intersection and Union of Sets
- The Types of Probability - Secondary 1 and 2
- Types of Events - Secondary 1 and 2

Modes of Representation and the Enumeration of Possible Outcomes

1. A bubble tea counter in Rosemont prepares one drink at random for a taste test. It picks one tea — green, black, taro or mango — then one topping — tapioca, jelly or none — then one size — small or large.

- a) How many different drinks are possible? Show the calculation.
- b) Explain why a two-column table is awkward for this experiment, and say which mode of representation you would use instead.
- c) List every possible drink that uses green tea, writing each one as a triple such as (green, jelly, large).

Probability

- 2.** A board game uses a spinner with 8 sectors, all the same size: 4 sectors say *advance 1*, 2 say *advance 2*, 1 says *advance 3* and 1 says *lose a turn*.
- a) Explain why the 8 sectors give 8 equally likely outcomes.
 - b) Find $P(\text{advance } 2)$ as a fraction in lowest terms, as a decimal and as a percentage.
 - c) Find the probability that the player advances at all.
 - d) A game lasts 200 spins. About how many times should a player expect *lose a turn*? Is that number guaranteed?

Random Experiments Where Order Matters and Where Order Does Not Matter

- 3.** A cycling club has five members: Alix, Bao, Camille, Dilan and Élodie. Use their initials A, B, C, D, E.
- a) Three of them are picked to ride a relay, and the order of the picking does not matter. List every possible team, in alphabetical order, and count them.
 - b) Instead, three of them are picked as first rider, second rider and third rider, so the order does matter. How many possibilities? Show the calculation.
 - c) Divide the answer to b) by the answer to a), and explain in words what that number counts.

Random Experiments With One or More Steps

4. A two-step game rolls a fair six-sided die numbered 1 to 6, then spins a spinner with four equal sectors numbered 1 to 4. The two numbers are added.

- a) How many equally likely results does the game have? Build a table showing the total for each of them.
- b) Find the probability that the total is exactly 5.
- c) Find the probability that the total is 9 or more.

Random Experiments With and Without Replacement

5. A jar holds 6 keychains: 4 blue (B_1, B_2, B_3, B_4) and 2 orange (O_1, O_2). Two keychains are taken.
- The first is *not* put back. Treating the two keychains as an unordered pair, how many pairs are possible, and what is $P(\text{both blue})$?
 - The first *is* put back and the results are written in order. How many results are possible now, and what is $P(\text{both blue})$?
 - Which version makes “both blue” more likely? Explain why, using the contents of the jar.

Sample Space

6. Three roommates — Ana, Ben and Coralie — draw lots for three different chores: dishes (D), recycling (R) and sweeping (S). Each person ends up with exactly one chore, and no chore is given twice.
- Write the sample space Ω , using a triple such as (D, R, S) to mean “Ana gets dishes, Ben gets recycling, Coralie gets sweeping”.
 - Explain why this is a draw *without* replacement, and how that shows in the size of Ω .
 - Find $P(\text{Ana gets the dishes})$ and $P(\text{Ben gets recycling and Coralie gets sweeping})$.

The Concept of “Or” and “And” in Probability

7. A bag holds 6 tiles: $A1$, $A2$, $A3$, $B1$, $B2$, $B3$. One tile is drawn at random. Let E be “the tile has the letter A ” and F be “the tile has an odd number”.

- a) List the outcomes of E and the outcomes of F .
- b) Find $P(E \text{ and } F)$.
- c) Find $P(E \text{ or } F)$, then check your answer with $P(E) + P(F) - P(E \text{ and } F)$.
- d) Give an event of this experiment that is incompatible with E .

The Intersection and Union of Sets

8. Thirty students were asked about three activities: skating (S), music lessons (M) and volunteering (V). The survey found 15 skaters, 12 in music lessons and 9 volunteers; 5 students do skating and music, 4 do skating and volunteering, 3 do music and volunteering, and 2 do all three. Each of those three pair counts includes the students who do all three activities.

- a) Draw a Venn diagram with three overlapping circles and write the correct number in each of its eight regions. Start from the centre.
- b) Find the probability that a randomly chosen student does exactly one of the three activities.
- c) Find the probability that a randomly chosen student does none of them.

The Types of Probability - Secondary 1 and 2

9. A thumbtack dropped on a table lands either point-up or point-down. Camille drops one 400 times and records 244 landings point-up.

- a) Find the experimental probability that the thumbtack lands point-up. Give it as a decimal and as a percentage.
- b) Camille's classmate says the theoretical probability must be $\frac{1}{2}$, "since there are two ways to land". Explain why no theoretical probability can be found for this experiment.
- c) Predict about how many point-up landings there would be in 1000 drops.

Types of Events - Secondary 1 and 2

- 10.** Twenty-four tiles numbered 1 to 24 are placed in a bag and one is drawn at random.
- a) For each event below, give its probability and name the most precise type that applies, choosing from *certain*, *impossible*, *elementary* and *probable*:
- A : the number is a multiple of 6; B : the number is greater than 24;
 C : the number is 13; D : the number is less than 25.
- b) Let E be “the number is odd”. Are A and E compatible? Justify.
- c) Is E complementary to “the number is even”? Check both conditions.
- d) Name an event that is incompatible with D , and explain why every such event must be impossible.

Synthesis — drawing on several sheets in this topic

11. A gym bin holds 6 balls: 2 blue (B_1, B_2) and 4 white ($W_1, \dots, W_4$). Two balls are taken out one after the other and the order is recorded.

- a) Explain why labelling the balls, rather than just calling them “blue” and “white”, is what makes the outcomes equally likely.
- b) Without replacement: how many results are in the sample space, and what is the probability of getting one ball of each colour?
- c) With replacement: answer the same two questions.
- d) In which version is “one of each colour” more likely? Explain why, using what is in the bin at the second draw.

Want the harder problems and the worked solutions?

The *Challenge Problems* set and the *Answer Keys* for Probability — with the reasoning written out in full — are available at tutorinmontreal.ca.