

Algebra — Relations and Functions

Name: _____

Date: _____

Concepts

- Algebra — Relations and Functions
- Finding the Equation of a Linear Function
- Graphing a Linear Function
- Solving Problems Involving Linear Functions
- The Inverse Variation Function (Inversely Proportional Situation)
- The Inverse of a Function
- The Inverse of the Linear Function
- The Piecewise Function
- The Properties of Functions
- The Properties of a Linear Function
- The Rate of Change (a) and the y-Intercept (b)
- The Role of Parameters in a Linear Function (theory)
- Types of Variables
- Zero- and First- Degree Polynomial Functions (Linear)

Algebra — Relations and Functions

1. In each situation below, name the independent variable and the dependent variable, then decide whether the dependent variable is a *function* of the independent one — that is, whether each value of the independent variable gives exactly one value of the dependent variable.

- a) Amélie is paid \$14 for each hour she works at a garden centre. We look at how much she earns.
- b) A guidance counsellor records, for every student in a class, the student's shoe size and the student's height.
- c) A snow-clearing truck travels at a steady 30 km/h. We look at the distance it has covered since it left the garage.

Finding the Equation of a Linear Function

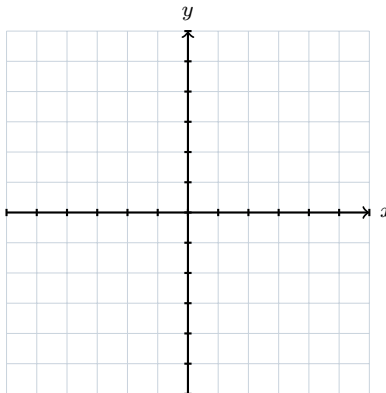
2. A pool is being filled by a hose. After 3 minutes it holds 46 litres, and after 8 minutes it holds 106 litres. The volume is a linear function of the time.

- Find the rate of change and say what it means here.
- Find the initial value and write the rule.
- How much water is in the pool after 15 minutes?

Graphing a Linear Function

3. On 1 March a snow bank at the corner of a street is 6 m high. It melts at a steady 1.5 m per week.

- Write the rule for the height h , in metres, after w weeks.
- Graph the function on the grid for w from 0 to 4.
- Read from your graph the week in which the snow bank disappears, and check it in the rule.



Solving Problems Involving Linear Functions

4. A skatepark offers two ways to pay. Option A costs \$8 for each visit. Option B costs \$50 for a season membership plus \$3 for each visit.

- a) Write a rule for the total cost of each option in terms of the number of visits v .
- b) For how many visits do the two options cost the same?
- c) Zack expects to go 14 times this season. Which option should he choose, and how much does he save?

The Inverse Variation Function (Inversely Proportional Situation)

5. A band rents a jam room for \$72 an evening and the members share the cost equally.

- a) Complete the table of what each member pays.

Members n	2	3	4	6	8	12
Each pays (\$)						

- b) Write the rule giving the amount c each member pays.
- c) No member wants to pay more than \$10. What is the smallest number of members that makes this possible?

The Inverse of a Function

6. A chairlift carries skiers up a hill. The table gives the altitude reached after t minutes.

Time t (min)	0	2	4	6
Altitude (m)	400	460	520	580

- a) Build the table of the inverse relation.
- b) Say in one sentence what the inverse tells a skier that the original does not.
- c) Use the inverse to find how long the lift takes to reach an altitude of 700 m.

The Inverse of the Linear Function

7. A photo lab charges $c = 0.25n + 4$ dollars to print n photos, where the \$4 is a fixed handling fee.

- a) Find the rule of the inverse, giving n in terms of c .
- b) How many photos can be printed for \$19?
- c) Verify your answer in the original rule.

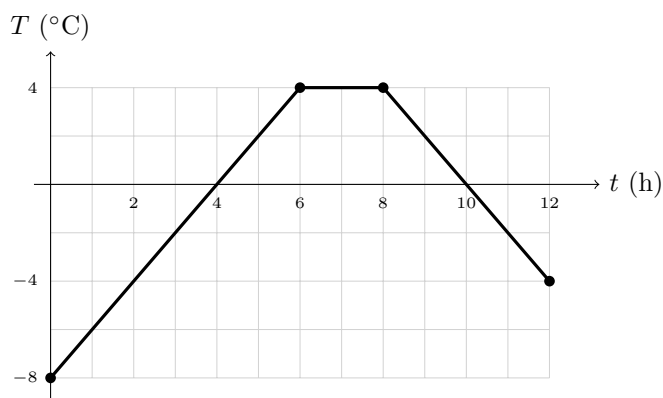
The Piecewise Function

8. Maëva cycles to her cousin's place. She rides at a steady 20 km/h for the first hour, then rests for half an hour, then rides at 24 km/h for the last half-hour.

- a) How far has she gone after 1 hour?
- b) What is her rate of change between 1 h and 1.5 h, and how would that part of the graph look?
- c) How far has she gone in total after 2 hours?

The Properties of Functions

9. The graph shows the outdoor temperature T , in degrees Celsius, during the first 12 hours of a January day.



- Give the domain and the range of the function in this context.
- Give the initial value, the maximum and the minimum, saying when each occurs.
- Over which time intervals is the temperature increasing, constant, decreasing?
- At what times is the temperature zero, and when is it above freezing?

The Properties of a Linear Function

10. A climbing gym sells a prepaid card worth \$240. Each visit takes \$12 off the card, so the value left after n visits is $f(n) = -12n + 240$.

- a) Give the initial value and the zero of the function, and say what each one means for the card holder.
- b) Is the function increasing or decreasing? Justify using the rate of change.
- c) Give the domain and the range in this context, remembering that a visit cannot be split in half.

The Rate of Change (a) and the y -Intercept (b)

11. A rain barrel is filling from a roof gutter during a steady shower.

Time t (min)	0	5	10	15
Volume V (L)	30	42	54	66

- a) Find the rate of change a and say what it means, with its units.
- b) Give the y -intercept b and say what it means, then write the rule.
- c) The barrel holds 90 L. When does it overflow?

The Role of Parameters in a Linear Function

12. Four lines are given by their rules:

$$y = 2x + 3, \quad y = 2x - 1, \quad y = -2x + 3, \quad y = 0.5x - 1.$$

Answer without drawing any of them, and justify each answer by naming the value in the rule you used.

- a) Which two lines are parallel?
- b) The four lines fall into two groups that cross the vertical axis at the same point. Give each group and the point it crosses at.
- c) Which line falls from left to right?
- d) Of the lines that rise, which rise most steeply?

Types of Variables

13. For each study, name the independent and the dependent variable, say whether each variable is qualitative or quantitative, and for the quantitative ones say whether they are discrete or continuous.

- a) A snow-clearing crew records the depth of snow that fell, in centimetres, and the time in hours needed to clear one street.
- b) A student council records each student's favourite winter sport and how many students chose it.
- c) A hockey team records the number of players on its roster and the total cost of the jerseys, at \$65 each.

Zero- and First-Degree Polynomial Functions (Linear)

14. For each rule, say whether it is a zero-degree or a first-degree polynomial function, and give its rate of change and its y -intercept.

- a) $f(x) = 9$
- b) $g(x) = -4x$
- c) $h(x) = 6 - 1.5x$
- d) The cost, in dollars, of x pastries at \$2.50 each, with no other charge.

Synthesis — drawing on several sheets in this topic

15. A diver begins an ascent 30 m below the surface and rises at a steady 2.5 m per minute. Depths below the surface are written as negative numbers, so the diver's position is $f(t) = 2.5t - 30$, where t is the time in minutes.

- a) Give the rate of change and the initial value, and say what each means for the diver.
- b) Find the rule of the inverse, giving the time in terms of the position.
- c) Use the inverse to find when the diver reaches the surface and when the diver passes the -12 m mark.
- d) Give the domain and the range of f for this ascent, and say on which interval f is negative.

Want the harder problems and the worked solutions?

The *Challenge Problems* set and the *Answer Keys* for Algebra — Relations and Functions — with the reasoning written out in full — are available at tutorinmontreal.ca.