Secondary 3 Mathematics · Practice Worksheet · tutorinmontreal.ca

Real Numbers and Number Sets

Name: _____ Date: _____

Concepts

- Absolute Value Notation - Secondary 1, 2 and 3
- Arithmetic (theory)
- Ascending and Descending Order
- Integers ($\mathbb{Z}$)
- Irrational Numbers ($\mathbb{R} \setminus \mathbb{Q}$ or $\mathbb{Q}'$)
- Natural Numbers ($\mathbb{N}$)
- Number Sets - Secondary 3 (theory)
- Ordering Irrational Numbers
- Ordering Real Numbers
- Rational Numbers ($\mathbb{Q}$)
- Real Numbers ($\mathbb{R}$)
- Representing Number Sets - Secondary 3
- The Common Denominator - Secondary 1, 2 and 3
- Writing Numbers (theory)

Absolute Value Notation — Secondary 1, 2 and 3

1. The absolute value of a number is its distance from zero on the number line, so it is never negative.

a) $|-9|$

b) $|4.5|$

c) $|-\frac{3}{4}|$

d) Two different numbers both have an absolute value of 12. Which two?

e) One January morning it is $-27\,^{\circ}\text{C}$ in Chibougamau and $-8\,^{\circ}\text{C}$ in Montréal. Which reading is further from $0\,^{\circ}\text{C}$, and by how much?

Arithmetic

2. Arithmetic is the branch of mathematics that studies numbers and the operations carried out on them.

a) Name the result of each of the four basic operations (the words, not any numbers).

b) In the fraction $\frac{45}{6}$, name the number written above the line and the one written below it.

c) Which operation does each of these phrases signal: “increased by”, “shared equally among”, “three times as many”, “how much more than”?

d) Compute $7 + 0$, 7×1 and 7×0. Which number leaves any addition unchanged, which leaves any multiplication unchanged, and which turns every product into zero?

Ascending and Descending Order

3.

a) Write in ascending order: -3.4, $\frac{7}{2}$, $-\frac{18}{5}$, 0, 3.25, -3.

b) Write in descending order: $\sqrt{16}$, $-\frac{9}{4}$, 2.5, -2.3, $\frac{15}{4}$.

c) Five snow depths measured on the same day are 42 cm, $\frac{3}{4}$ of a metre, 0.38 m, 59 cm and $\frac{1}{2}$ of a metre. Put them in ascending order, all expressed in centimetres.

Integers ($\mathbb{Z}$)

4. The integers are the natural numbers together with their opposites: $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$.

a) Which of these belong to $\mathbb{Z}$: -7, $\frac{8}{2}$, 0, $-\frac{5}{3}$, $\sqrt{49}$, 4.0, -2.5?

b) Write the opposite of -13, and the opposite of 0.

c) A downtown parking garage numbers its levels with integers: level $+3$ is three floors above the ground and level -3 is three floors below. A delivery van starts on level -4 and climbs 7 levels. Which level does it reach?

d) How many integers lie strictly between -5 and 3?

Irrational Numbers

5. An irrational number cannot be written as a quotient of two integers. In decimal form, its expansion never ends and never falls into a repeating block.

a) Which of these are irrational: $\sqrt{25}$, $\sqrt{10}$, π, $0.333\ldots$ (the 3 repeating forever), $\frac{22}{7}$, $-\sqrt{2}$, $1.010010001\ldots$ (one extra zero in each group)?

b) $\sqrt{25}$ and $\sqrt{10}$ end up in different groups. Explain, in one sentence, what separates them.

c) Give $\sqrt{10}$ to two decimal places, and explain why the value you wrote is not equal to $\sqrt{10}$.

Natural Numbers ($\mathbb{N}$)

6. The natural numbers are the counting numbers together with zero: $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$.

a) Which of 0, -6, $\frac{12}{4}$, $\frac{7}{2}$, $\sqrt{81}$, 15, 2.0 belong to $\mathbb{N}$?

b) Which of these quantities must be a natural number, and which need not be: the number of buses booked for a school trip; the temperature outside in °C; the number of goals scored in a hockey game; the mass of a backpack in kilograms; a page number in a book?

c) What is the smallest natural number? Explain why there is no largest one.

Number Sets — Secondary 3

7. The number sets studied in Secondary 3 sit inside one another: $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$. The irrational numbers are the real numbers that are *not* rational.

a) Say in words what the chain $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$ claims.

b) For each number, name the smallest of the four sets that contains it: -8, $\frac{3}{5}$, $\sqrt{7}$, 12, -4.25, π, $\sqrt{36}$.

c) True or false, with a reason: every integer is a rational number.

d) Can a number be both rational and irrational? Explain.

Ordering Irrational Numbers

8.

a) Without a calculator, place each of $\sqrt{3}$, $\sqrt{18}$, $\sqrt{27}$ and $\sqrt{50}$ between two consecutive whole numbers.

b) Use a) to write those four numbers in ascending order.

c) Write π, $\sqrt{8}$, $\sqrt{11}$ and 3.2 in ascending order, and state the comparison you used for each decision.

Ordering Real Numbers

9.

a) Write in ascending order: $-\frac{7}{2}$, $\sqrt{5}$, -3.45, $\frac{9}{4}$, 0, $-\sqrt{9}$, 2.3.

b) Which of the seven numbers are irrational?

c) Explain why comparing $\sqrt{5}$ with $\frac{9}{4}$ takes more care than comparing -3.45 with 0, and show the comparison that settles it.

Rational Numbers ($\mathbb{Q}$)

10. A rational number is one that can be written as $\frac{a}{b}$ with a and b integers and $b \neq 0$. In decimal form, its expansion either stops or eventually repeats.

a) Write each of these as a quotient of two integers: 0.75, -6, $2\frac{3}{8}$, $0.444\ldots$, 12%.

b) Write $\frac{5}{8}$ and $\frac{4}{11}$ in decimal form, and say which one stops and which one repeats.

c) Is $\frac{0}{7}$ a rational number? Is $\frac{7}{0}$? Explain both answers.

Real Numbers ($\mathbb{R}$)

11. The real numbers are the rational numbers together with the irrational numbers — every number that can be placed on the number line.

a) Sort into rational and irrational: $\sqrt{64}$, $-\frac{11}{6}$, $\sqrt{6}$, $0.818181\ldots$, 3π, -5.2.

b) Give one example each of a real number that is a natural number, an integer that is not natural, a rational number that is not an integer, and an irrational number.

c) Give one rational number and one irrational number lying strictly between 2.7 and 2.8, and say how you know each is what you claim.

Representing Number Sets — Secondary 3

12. A set of numbers can be given in words, drawn on a number line, or written in interval notation. A square bracket turned towards the interval includes the endpoint; a reversed square bracket excludes it. Infinity is always excluded.

a) Write in interval notation: all real numbers greater than or equal to -2 and less than 6.

b) Write in interval notation: all real numbers less than 4.

c) Describe in words the set $]-\infty, -1]$.

d) Which of -2, 0, 5.9, 6 belong to $[-2, 6[$?

e) Write $]3, +\infty[$ as an inequality.

The Common Denominator — Secondary 1, 2 and 3

13.

a) Write $\frac{5}{6}$, $\frac{7}{9}$ and $\frac{11}{12}$ over their least common denominator, then put the three in ascending order.

b) Compute $\frac{5}{6} + \frac{7}{9} - \frac{11}{12}$, giving the answer in lowest terms.

c) A canoe route is shared by three paddlers: Léa covers $\frac{3}{8}$ of it and Noah covers $\frac{1}{3}$. What fraction does Zoé, who covers the rest, paddle?

d) In one sentence, say why fractions need a common denominator to be added but not to be multiplied.

Writing Numbers

14. The same number can be written in several notations — fractional, decimal, percentage and scientific — and choosing well is part of communicating clearly.

a) Complete the table; each row holds one number written three ways.

Fraction	Decimal	Percentage
$\frac{3}{4}$		
	0.06	
		145%

b) Write 0.000082 in scientific notation, and write 4.7×10^5 in decimal notation.

c) Give one situation where a percentage communicates better than a fraction, and one where a fraction communicates better than a decimal.

Synthesis — drawing on several sheets in this topic

15. A weather station's log holds five real numbers:

$$A = -\tfrac{17}{4}, \qquad B = \sqrt{20}, \qquad C = -4.2, \qquad D = 0.666\ldots, \qquad E = \tfrac{9}{2}.$$

a) Say which of the five are rational and which are irrational, with a reason for each irrational one.

b) Write all five in ascending order.

c) Which of the five is furthest from zero? Use absolute value to justify.

d) Write, in interval notation, the shortest interval whose two endpoints are values from the list and which contains all five.