

Algebra - Algebraic Expressions

Name: _____

Date: _____

Concepts

- Adding and Subtracting Rational Expressions
- Completing the Square
- Dividing Rational Expressions
- Dividing an Algebraic Expression by a Binomial
- Exponentiation in Algebraic Expressions
- Factoring Out a Greatest Common Factor
- Factoring Polynomials (theory)
- Factoring a Trinomial Using the Quadratic Formula
- Factoring by Grouping
- Multiplying Algebraic Expressions - Secondary 3 and 4
- Multiplying Rational Expressions
- Operations on Algebraic Expressions (theory)
- Simplifying Rational Expressions
- The Difference of Squares
- The Perfect Square Trinomial
- The Product-Sum Technique

Adding and Subtracting Rational Expressions

1. Write as a single rational expression in simplified form and state the restrictions on x :

$$\frac{5}{x+2} + \frac{3}{x-1}.$$

Completing the Square

2. Write $f(x) = x^2 - 10x + 31$ in the standard form $a(x-h)^2 + k$ and state the coordinates of its vertex.

Dividing Rational Expressions

3. Simplify and state the restrictions:

$$\frac{x^2 - 9}{4x} \div \frac{x + 3}{8x^2}.$$

Dividing an Algebraic Expression by a Binomial

4. Carry out the division $(2x^3 - x^2 - 16x + 3) \div (x - 3)$ and state the quotient and the remainder.

Exponentiation in Algebraic Expressions

5. Simplify, writing the result with positive exponents only:

$$\left(\frac{2x^3y^{-2}}{5x^{-1}y^4} \right)^2, \quad x \neq 0, y \neq 0.$$

Factoring Out a Greatest Common Factor

6. Factor completely: $12a^3b - 18a^2b^2 + 30a^2b$.

Factoring Polynomials

7. For each polynomial below, name the factoring technique that should be applied *first*, then give the complete factorization.

a) $x^2 - 49y^2$

b) $3x^3 - 12x$

c) $x^2 + 10x + 25$

d) $2ax + 6a + bx + 3b$

Factoring a Trinomial Using the Quadratic Formula

8. Use the quadratic formula to factor $6x^2 - 7x - 20$ over the integers.

Factoring by Grouping

9. Factor by grouping: $3xy - 6x + 5y - 10$.

Multiplying Algebraic Expressions — Secondary 3 and 4

10. Expand and simplify: $(3x - 4)(2x + 5) - (x - 3)^2$.

Multiplying Rational Expressions

11. Simplify and state the restrictions:

$$\frac{x^2 - 25}{3x + 9} \cdot \frac{x^2 + 4x + 3}{x^2 - 4x - 5}.$$

Operations on Algebraic Expressions

12. Decide whether each statement is true for *all* polynomials concerned. Justify a true statement with an argument and a false one with an explicit counterexample.

- a) The product of two binomials is always a trinomial.
- b) The sum of two polynomials of degree 3 is always a polynomial of degree 3.
- c) The product of a polynomial of degree 2 and a polynomial of degree 3 always has degree 5.

Simplifying Rational Expressions

- 13.** Simplify $\frac{3x^2 - 12}{x^2 + 4x + 4}$ and state the restriction.

The Difference of Squares

- 14.** Factor completely: $50x^2 - 8y^2$.

The Perfect Square Trinomial

- 15.** Factor $9x^2 - 42xy + 49y^2$.

The Product-Sum Technique

- 16.** Factor $6x^2 + 11x - 35$ using the product-sum technique.

Synthesis — drawing on several sheets in this topic

17. An outdoor skating rink is rectangular. Its surface area, in m^2 , is $A(x) = 6x^2 + x - 12$ and its width is $(3x - 4)$ m.

- a) Determine an expression for the length of the rink.
- b) A walkway of uniform width 1 m is poured all around the rink. Express the area of the walkway alone as a simplified polynomial.
- c) The walkway required exactly 62 m^2 of concrete. Find the dimensions of the rink.

Want the harder problems and the worked solutions?

The *Challenge Problems* set and the *Answer Keys* for Algebra - Algebraic Expressions — with the reasoning written out in full — are available at tutorinmontreal.ca.