

Step Functions (Greatest Integer)

Name: _____

Date: _____

Concepts

- Finding the Rule of a Step Function (Greatest Integer Function)
 - Graphing a Step Function (Greatest Integer Function)
 - Solving Problems Involving the Step Function (the Greatest Integer Function)
 - The Inverse of the Step Function (Greatest Integer Function) (theory)
 - The Properties of the Step Function (Greatest Integer Function)
 - The Role of the Parameters in a Step Function (Greatest Integer Function) (theory)
 - The Step Function (Greatest Integer Function)
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Finding the Rule of a Step Function (Greatest Integer Function)

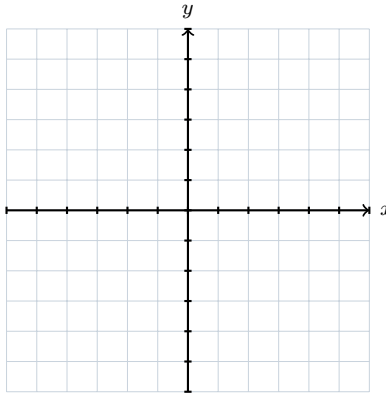
1. Consider $f(x) = -2 \left\lfloor \frac{1}{3}(x - 1) \right\rfloor + 5$.

a) Compute $f(4)$, $f(6.5)$ and $f(-2)$.

b) State the length of each step, the size of the jump between two consecutive steps, and whether the solid (closed) endpoint of each step is on the left or on the right.

Graphing a Step Function (Greatest Integer Function)

2. Sketch $f(x) = \left\lfloor \frac{1}{2}x \right\rfloor$ on the interval $[-4, 6[$, marking clearly which endpoint of each step is closed. Give the value of $f(3.7)$ and of $f(-0.5)$.



Solving Problems Involving the Step Function (the Greatest Integer Function)

3. A riverside outfitter rents kayaks and charges \$9 for each started 30-minute period. For a rental of t minutes ($t > 0$) the cost in dollars is

$$C(t) = 9 \left\lceil \frac{t}{30} \right\rceil.$$

- a) What does a 74-minute rental cost?
- b) What is the longest rental that still costs \$27?

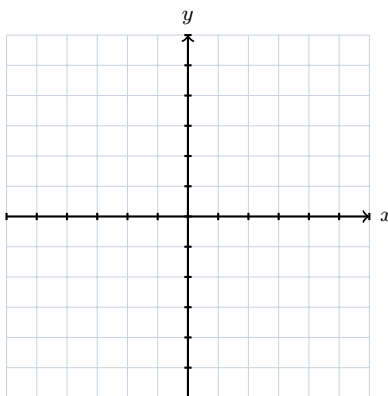
The Inverse of the Step Function (Greatest Integer Function)

4. Consider the greatest integer function $f(x) = \lfloor x \rfloor$ with domain $\mathbb{R}$. Explain why its inverse is *not* a function, and describe the inverse relation precisely (which elements it starts from, and what each one is paired with).

The Properties of the Step Function (Greatest Integer Function)

5. Let $f(x) = 2\lfloor x - 1 \rfloor - 1$.

- Evaluate $f(4.6)$ and $f(-2.3)$.
- Give the length of each step and the vertical distance between two consecutive steps.
- State the range of f .
- Sketch f for $x \in [-1, 4]$ on the grid, showing clearly which endpoint of each step is included.



The Role of the Parameters in a Step Function (Greatest Integer Function)

6. Consider $f(x) = -4\lfloor 3(x - 1) \rfloor + 5$. Without drawing the graph, state the length of each step, the vertical distance between two consecutive steps, and whether the steps rise or fall as x increases. Say which endpoint of each step is included, and justify your answer using the signs of a and b .

The Step Function (Greatest Integer Function)

7. At a train station, the fee in dollars for leaving a bag at the luggage counter for t hours is $F(t) = 2\lfloor t \rfloor + 5$, for $0 \leq t < 12$.

- a) Find $F(3.9)$ and $F(4)$.
- b) For how long may a bag be left if exactly \$13 is paid? Give an interval.

Want the harder problems and the worked solutions?

The *Challenge Problems* set and the *Answer Keys* for Step Functions (Greatest Integer) — with the reasoning written out in full — are available at tutorinmontreal.ca.