

Analytic Geometry

Name: _____

Date: _____

Concepts

- Analytic Geometry (theory)
- Forms of the Equation of a Line
- Proofs in Analytic Geometry (theory)
- Straight Lines and Half-Planes (theory)
- The Distance Between 2 Points
- The Distance Between Two Parallel Lines
- The Distance From a Point to a Line on a Cartesian Plane
- The Division Point and Midpoint of a Segment
- The Equation of a Line From the Coordinates or Slope
- The Graph of a Line
- The Half-Plane and the Solution Set
- The Relative Position of Two Lines
- The Slope of a Line

Analytic Geometry

1. Analytic geometry answers geometric questions with coordinates and algebra. For each situation below, name the analytic tool you would use (a formula or a condition on slopes), and say what the tool produces — a number, a pair of coordinates, or a yes/no verdict.

- a) Deciding whether two segments of a roof truss meet at a right angle.
- b) Locating the point three-fifths of the way along a segment from A to B .
- c) Showing that a triangle drawn on a plan is isosceles.
- d) Deciding whether a house at $(4, -1)$ lies on the protected side of a straight shoreline.

Forms of the Equation of a Line

2. A conveyor belt is modelled by the line $3x + 4y - 24 = 0$.

- a) Rewrite it in functional form and state the slope.
- b) Rewrite it in symmetric form and state both intercepts.

Proofs in Analytic Geometry

3. To prove that *the diagonals of a rhombus are perpendicular*, a student draws the rhombus with vertices $(0, 0)$, $(5, 0)$, $(8, 4)$ and $(3, 4)$, checks that all four sides measure 5 units, computes the slopes of the two diagonals as $\frac{1}{2}$ and -2 , notes that the product is -1 , and writes “proved”. Explain why this is not a proof, and describe precisely how the coordinates should be chosen so that the same computation does establish the general statement.

Straight Lines and Half-Planes

4. The line $2x - 3y = 6$ cuts the plane into two half-planes. For each set below, say whether it is a line, an open half-plane, a closed half-plane, or none of these, and justify in one sentence.

- a) $\{(x, y) \mid 2x - 3y > 6\}$
- b) $\{(x, y) \mid 2x - 3y = 6\}$
- c) $\{(x, y) \mid |2x - 3y| \geq 6\}$
- d) $\{(x, y) \mid y \leq 4\}$

The Distance Between 2 Points

5. On a climbing wall, coordinates are given in metres. An anchor bolt sits at $A(-3, 7)$ and another at $B(9, 2)$. How long a rope is needed to run straight from one bolt to the other?

The Distance Between Two Parallel Lines

6. Find the distance between the parallel lines $y = \frac{3}{4}x + 2$ and $y = \frac{3}{4}x - 3$.

The Distance From a Point to a Line on a Cartesian Plane

7. Find the distance from the point $P(7, -2)$ to the line $4x - 3y + 1 = 0$.

The Division Point and Midpoint of a Segment

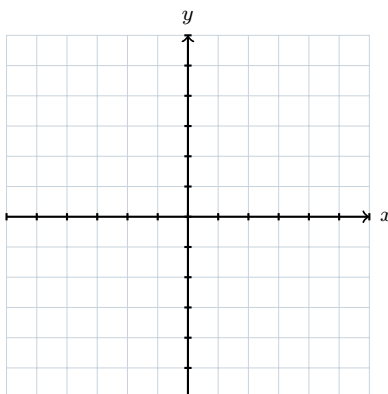
8. A straight ski trail runs from the chalet $A(-6, 2)$ to the summit $B(6, 10)$ (units in hundreds of metres).
- a) Find the midpoint M of $\overline{AB}$.
 - b) A rest bench is placed one quarter of the way from A to B . Find its coordinates.

The Equation of a Line From the Coordinates or Slope

9. Find the equation, in functional form, of the line passing through $(-2, 11)$ and $(4, -7)$.

The Graph of a Line

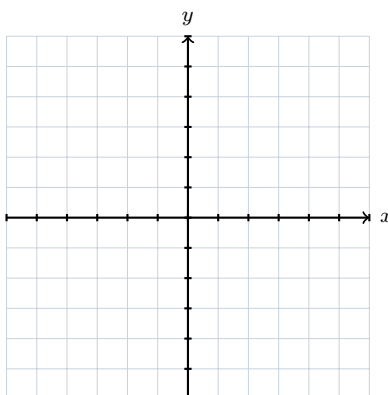
10. Draw the line $2x + 3y = 12$ on the grid below by first plotting its two intercepts, then state its slope.



The Half-Plane and the Solution Set

11. Consider the inequality $2x - y \leq 4$.

- Give the equation of the boundary line and say whether it is drawn solid or dashed.
- Use a test point to decide which side is shaded, then sketch the solution set on the grid.



The Relative Position of Two Lines

12. Three lines are given: $\ell_1 : 3x - 6y + 5 = 0$, $\ell_2 : y = \frac{1}{2}x - 4$ and $\ell_3 : 2x + y = 7$. Determine the relative position of each of the three pairs (ℓ_1, ℓ_2) , (ℓ_1, ℓ_3) and (ℓ_2, ℓ_3) .

The Slope of a Line

13. A wheelchair ramp rises 0.6 m over a horizontal run of 7.2 m.

- a) Give its slope as a fraction in lowest terms.
- b) Express that slope as a percent, to the nearest hundredth.

Synthesis — drawing on several sheets in this topic

14. On a site plan graduated in metres, a straight drainage pipe runs from $A(-4, -2)$ to $B(8, 4)$. A maintenance hatch H is installed on the pipe, two thirds of the way from A to B . A protected oak stands at $T(0, 9)$.

- a) Find the coordinates of H and the total length of the pipe, to the nearest hundredth of a metre.
- b) By-law: no tree may be closer than 6 m to a pipe. Does the oak comply? Support your answer with a computation.

Want the harder problems and the worked solutions?

The *Challenge Problems* set and the *Answer Keys* for Analytic Geometry — with the reasoning written out in full — are available at tutorinmontreal.ca.