

Algebra - Algebraic Expressions

Name: _____

Date: _____

Concepts

- Adding and Subtracting Rational Expressions
- Dividing Rational Expressions
- Factoring Polynomials (theory)
- Factoring a Trinomial Using the Quadratic Formula
- Multiplying Rational Expressions
- Operations on Algebraic Expressions (theory)
- Simplifying Rational Expressions
- The Difference of Squares

Adding and Subtracting Rational Expressions

1. Write as a single rational expression in lowest terms and state the restrictions on x :

$$\frac{5}{x-3} - \frac{2}{x+4}.$$

Dividing Rational Expressions

2. Simplify and state the restrictions on x :

$$\frac{x^2 - 9}{2x + 8} \div \frac{x + 3}{x^2 - 16}.$$

Factoring Polynomials

3. Four polynomials are given.

$$P_1 = 4x^2 - 25y^2, \quad P_2 = x^2 + 6x + 13, \quad P_3 = 3x^3 + 6x^2 - 2x - 4, \quad P_4 = 2x^2 + 7x - 15.$$

Exactly one of them cannot be factored over $\mathbb{R}$. Identify it and justify your choice. For each of the other three, name the technique that applies and give the complete factorization.

Factoring a Trinomial Using the Quadratic Formula

4. Use the quadratic formula to factor $6x^2 - 7x - 20$ completely.

Multiplying Rational Expressions

5. Simplify and state the restrictions on x :

$$\frac{x^2 + 7x + 10}{3x - 9} \cdot \frac{x^2 - 5x + 6}{x^2 - 4}.$$

Operations on Algebraic Expressions

6. Marc states two rules about polynomials in one variable with real coefficients:

- a) “The sum of two polynomials of degree 3 always has degree 3.”
- b) “The product of two polynomials of degree 3 always has degree 6.”

For each statement, decide whether it is true. Prove the true one and disprove the false one with an explicit counterexample.

Simplifying Rational Expressions

7. Reduce to lowest terms and state the restrictions on x :

$$\frac{3x^2 - 27}{x^2 + x - 12}.$$

The Difference of Squares

8. Factor completely: $81a^4 - 16b^4$.

Synthesis — drawing on several sheets in this topic

9. A ferry shuttles between two docks 48 km apart on a river. In still water it travels at v km/h; the current runs at a steady 4 km/h, so the ferry travels downstream on the way out and upstream on the way back.

- a) Write the total round-trip time as a sum of two rational expressions in v , then combine and simplify it into a single rational expression.
- b) State the values of v for which the model makes sense, and explain why.
- c) The round trip takes exactly 5 hours. Determine the ferry's still-water speed.

Want the harder problems and the worked solutions?

The *Challenge Problems* set and the *Answer Keys* for Algebra - Algebraic Expressions — with the reasoning written out in full — are available at tutorinmontreal.ca.