Secondary 5 Mathematics · Practice Worksheet · tutorinmontreal.ca

Algebra - Equations and Inequalities

Name: ______________________________ Date: ______________

Concepts

- Optimization (theory)
- Representing Inequalities on a Cartesian Plane
- Representing a Solution Set
- Solving Algebraic Inequalities
- Solving Equations and Inequalities
- Solving a Cosine Equation or Inequality
- Solving a Greatest Integer Equation
- Solving a Logarithmic Equation or Inequality
- Solving a Rational Equation or Inequality
- Solving a Second Degree Equation or Inequality
- Solving a Sine Equation or Inequality
- Solving a Square Root Equation or Inequality
- Solving a Tangent Equation or Inequality
- Solving a Trigonometric Equation or Inequality
- Solving an Equation or Inequality Containing an Absolute Value
- Solving an Exponential Equation or Inequality
- Solving an Optimization Problem
- Systems of Inequalities and the Polygon of Constraints

Optimization

1. A drone-delivery start-up plans x short routes and y long routes for a single day. Its planning sheet lists three lines:

a) $2x + 5y \leq 40$ (battery-hours available)

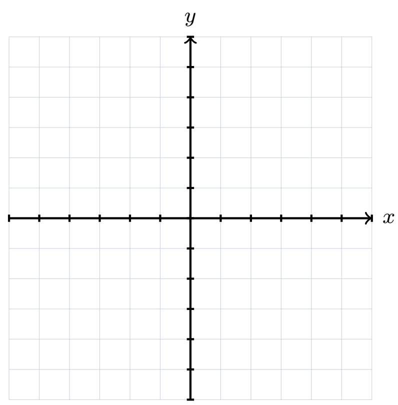
b) $P = 7x + 12y$ (profit, in dollars)

c) $x \geq 0,\ y \geq 0$

Which of these three lines belong to the constraint system and which one is the function to optimize? Then explain, using the shape of the level curves of P, why the maximum profit can never be reached at a point strictly inside a bounded polygon of constraints.

Representing Inequalities on a Cartesian Plane

2. Represent the solution set of $3x - 2y \leq 12$ on the Cartesian plane below. State the coordinates of the two intercepts of the boundary, say whether the boundary line is solid or dashed, and give the test point you used.



Representing a Solution Set

3. A greenhouse controller logs the difference x, in degrees Celsius, between the air temperature and the target temperature. The controller stays idle when x is at least -5 but less than 3, and it raises an alarm when x is greater than 8. Let S be the set of readings that are idle *or* alarming. Express S

a) in interval notation, using the Québec convention for open ends;

b) in set-builder notation.

Solving Algebraic Inequalities

4. Solve $5 - 3(x - 2) \geq 2x + 1$ and write the solution set in interval notation.

Solving Equations and Inequalities

5. Solve $\dfrac{2x-1}{3} - \dfrac{x+4}{2} = \dfrac{1}{6}$.

Solving a Cosine Equation or Inequality

6. Solve $2\cos x + 1 = 0$ for $x \in [0,\, 2\pi]$.

Solving a Greatest Integer Equation

7. Solve $\lfloor 2x-3 \rfloor = 5$ and give the solution set in interval notation.

Solving a Logarithmic Equation or Inequality

8. Solve $\log_3(2x-1) = 4$.

Solving a Rational Equation or Inequality

9. Solve $\dfrac{3}{x-2} = \dfrac{5}{x+4}$, stating the restrictions on x.

Solving a Second Degree Equation or Inequality

10. Solve the inequality $3x^2 + 2x - 8 > 0$ and write the solution set in interval notation.

Solving a Sine Equation or Inequality

11. Solve $3\sin(2x) + 1 = 2.5$ for $x \in [0, 2\pi]$.

Solving a Square Root Equation or Inequality

12. Solve $-2\sqrt{x+3} + 9 = 3$.

Solving a Tangent Equation or Inequality

13. Solve $\tan\left(x - \frac{\pi}{4}\right) = 1$ for $x \in [0, 2\pi[$.

Solving a Trigonometric Equation or Inequality

14. Solve $2\sin^2 x + \sin x - 1 = 0$ for $x \in [0, 2\pi[$.

Solving an Equation or Inequality Containing an Absolute Value

15. Solve $3|x-2|-4=8$.

Solving an Exponential Equation or Inequality

16. Solve $2 \cdot 3^{x+1} = 54$.

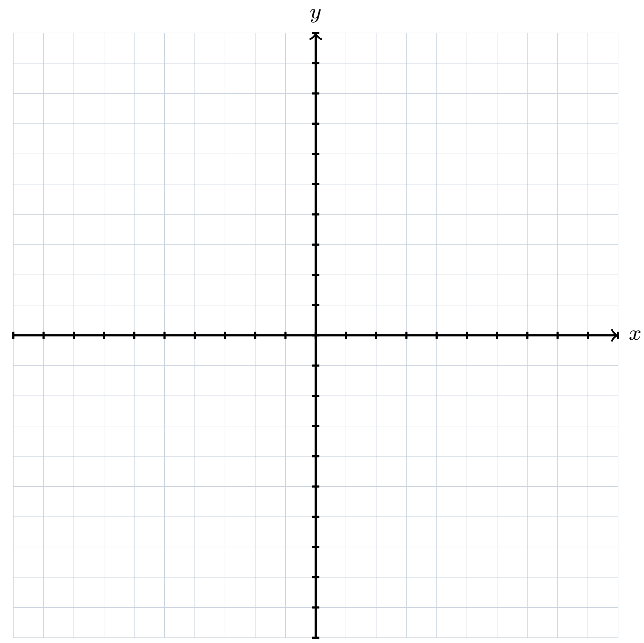
Solving an Optimization Problem

17. The polygon of constraints of a problem is the convex pentagon whose vertices are $(1,2)$, $(1,9)$, $(6,7)$, $(8,3)$ and $(5,1)$. The objective function is $Z = 4x + 5y$. Find the maximum and the minimum of Z on this polygon, and state where each occurs.

Systems of Inequalities and the Polygon of Constraints

18. Graph the system below and give the coordinates of all vertices of the polygon of constraints.

$$x \geq 0, \qquad y \geq 0, \qquad x + y \leq 10, \qquad 2x + y \leq 14$$



Synthesis — drawing on several sheets in this topic

19. A workshop makes two models of kite. A *box kite* uses 3 m of ripstop fabric and 1 h of sewing; a *delta kite* uses 2 m of fabric and 2 h of sewing. This month the workshop has 64 m of fabric and 40 h of sewing time, and a store has already ordered at least 5 delta kites. The profit is $9 per box kite and $12 per delta kite.

a) Define the variables and write the system of inequalities.

b) Determine the vertices of the polygon of constraints.

c) How many kites of each model should be made to maximize the profit, and what is that profit?