Secondary 5 Mathematics · Practice Worksheet · tutorinmontreal.ca

# Rational Functions

Name: _____ Date: _____

## Concepts

- Finding the Rule of a Rational Function
- Graphing a Rational Function
- Properties of the Rational Function
- Solving Problems Involving Rational Functions
- Switching from the Standard to the General Form of a Rational Function
- The Inverse of the Rational Function
- The Rational Function (theory)
- The Role of the Parameters in a Rational Function in Standard Form

### Finding the Rule of a Rational Function

**1.** A rational function has the asymptotes $x = 2$ and $y = -3$, and its curve passes through the point $(4, 1)$.

**a)** Find its rule in the form $f(x) = \dfrac{a}{x - h} + k$.

**b)** Give the coordinates of the two intercepts of the curve.

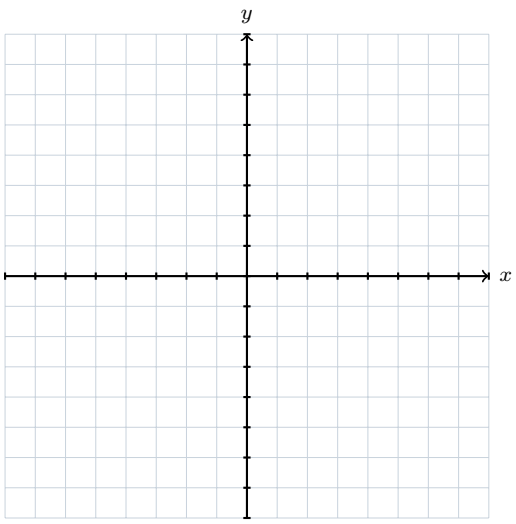
### Graphing a Rational Function

**2.** Consider $f(x) = \dfrac{6}{x-1} + 2$.

**a)** Give the equations of the two asymptotes and the coordinates of the two intercepts.

**b)** Sketch both branches on the grid below, using at least two extra plotted points per branch.

**c)** Give the domain and the range of $f$.




### Properties of the Rational Function

**3.** Let $f(x) = \dfrac{4}{x-3} + 2$.

**a)** Give the two asymptotes, the domain and the range.

**b)** Find the zero and the $y$-intercept.

**c)** Give the intervals where $f$ is positive and where it is negative.

### Solving Problems Involving Rational Functions

**4.** A robotics club buys a 3D printer for \$1800; each printed part then costs \$4 in filament. After $n$ parts, the average cost per part is

$$C(n) = \frac{1800 + 4n}{n}, \qquad n > 0.$$

**a)** What is the average cost per part after 150 parts?

**b)** How many parts must the club print for the average cost to drop below \$10?

**c)** What is the horizontal asymptote of $C$, and what does it mean here?

### Switching from the Standard to the General Form of a Rational Function

**5.**

**a)** Write $f(x) = \dfrac{3}{x-4} + 5$ in the general form $\dfrac{ax+b}{cx+d}$.

**b)** Write $g(x) = \dfrac{2x+11}{x+3}$ in standard form and give its asymptotes.

### The Inverse of the Rational Function

**6.** Let $f(x) = \dfrac{3}{x-4} + 2$. Find the rule of $f^{-1}$ and give the equations of the asymptotes of $f$ and of $f^{-1}$.

### The Rational Function

**7.** Here "rational function" means a function whose rule is a quotient of polynomials in which the variable actually appears in the denominator. For each rule, state whether it defines a rational function, and whether its graph has a vertical asymptote. Justify.

**a)** $a(x) = \dfrac{5}{2x-7}$

**b)** $b(x) = \dfrac{x^2-4}{x-2}$

**c)** $c(x) = \dfrac{x+1}{4}$

### The Role of the Parameters in a Rational Function in Standard Form

**8.** Let $f(x) = \dfrac{-4}{x+3} + 2$.

**a)** Give the equations of the two asymptotes, the domain and the range.

**b)** Is each branch increasing or decreasing? Justify with the sign of $a$.

**c)** Find the zero of $f$.