Secondary 5 Mathematics · Practice Worksheet · tutorinmontreal.ca

Step Functions (Greatest Integer)

Name: _____ Date: _____

Concepts

- Finding the Rule of a Step Function (Greatest Integer Function)
- Graphing a Step Function (Greatest Integer Function)
- Solving Problems Involving the Step Function (the Greatest Integer Function)
- The Inverse of the Step Function (Greatest Integer Function) (theory)
- The Properties of the Step Function (Greatest Integer Function)
- The Role of the Parameters in a Step Function (Greatest Integer Function) (theory)
- The Step Function (Greatest Integer Function)

Finding the Rule of a Step Function (Greatest Integer Function)

1. The graph of a step function is made of horizontal segments of length 2 whose heights increase by 3 from one segment to the next. The segment covering $4 \leq x < 6$ sits at the height $y = 5$, and every segment includes its *left* endpoint (closed dot on the left, open dot on the right).

a) Find a rule of the form $f(x) = a\lfloor b(x - h)\rfloor + k$.

b) Evaluate $f(11.4)$ and $f(3.5)$.

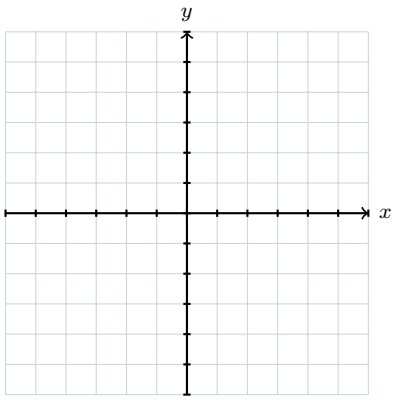
Graphing a Step Function (Greatest Integer Function)

2. Consider $f(x) = -2\lfloor x + 1 \rfloor + 3$, restricted to $-2 \leq x < 4$.

a) Give the length and the height of each step, and say which endpoint of each step is closed.

b) Sketch the graph on the grid below.

c) Evaluate $f(2.5)$ and $f(-0.5)$.



Solving Problems Involving the Step Function (the Greatest Integer Function)

3. A ferry carries at most 24 vehicles per crossing. The number of crossings needed to move n vehicles ($n \geq 1$) is

$$N(n) = \left\lfloor \frac{n-1}{24} \right\rfloor + 1.$$

a) How many crossings are needed for 100 vehicles?

b) For which numbers of vehicles are exactly 7 crossings needed?

The Inverse of the Step Function (Greatest Integer Function)

4. Let $f(x) = 3\lfloor x - 1 \rfloor + 2$.

a) Find every value of x whose image is 5.

b) Deduce why the inverse of a step function is never a function.

c) Describe the set of values on which the inverse relation of f is defined.

The Properties of the Step Function (Greatest Integer Function)

5. Let $f(x) = -2\lfloor x + 3 \rfloor + 1$.

a) Evaluate $f(0)$, $f(-1.5)$ and $f(2.7)$.

b) Give the length and the height of each step, and say whether f is increasing or decreasing.

c) Describe the range of f.

The Role of the Parameters in a Step Function (Greatest Integer Function)

6. Consider $f(x) = -3\left\lfloor \frac{1}{2}(x-1) \right\rfloor + 4$. Without graphing, determine:

a) the length of each step and the vertical distance between two consecutive steps;

b) whether the steps go up or down as x increases;

c) whether the solid (included) endpoint of each step is at its left or its right end — justify, then confirm your answer by computing $f(1)$ and $f(3)$.

The Step Function (Greatest Integer Function)

7. Evaluate $\lfloor -3.2 \rfloor$, $\lfloor 4 \rfloor$ and $\left\lfloor -\frac{7}{2} \right\rfloor$. Then solve $\left\lfloor \frac{x}{2} \right\rfloor = 3$ and write the solution set in interval notation.