

Analytic Geometry

Name: _____

Date: _____

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- Conics
- Dilations on a Cartesian Plane
- Ellipse (Conic)
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Analytic Geometry

1. Analytic geometry describes a geometric object by an equation on the Cartesian plane. For each equation below, name the curve it represents (line, circle, ellipse, parabola or hyperbola) and give the one feature of the equation that settles your choice.

a) $x^2 + y^2 - 6x + 4y = 12$

b) $\frac{(x+1)^2}{9} - \frac{(y-2)^2}{4} = 1$

c) $y = -2(x-3)^2 + 5$

d) $3x - 5y = 15$

e) $\frac{x^2}{25} + \frac{y^2}{16} = 1$

Circle (Conic)

2. A city park has a circular splash pad whose centre, on the park map, is the point $(-2, 5)$ and whose radius is 6 m.

- a) Write the equation of the edge of the splash pad.
- b) A drinking fountain stands at $(3, 7)$. Is it inside the splash pad, on its edge, or outside? Justify with a computation.

Conics

3. Write $9x^2 + 4y^2 - 54x + 8y + 49 = 0$ in standard form. Name the conic, and give its centre and the length of each of its axes.

Dilations on a Cartesian Plane

4. Triangle ABC has vertices $A(-1, 2)$, $B(4, 0)$ and $C(2, -3)$. It undergoes the dilation $h_{(O, 3)}$ centred at the origin with ratio 3.

- a) Give the coordinates of A' , B' and C' .
- b) By what factor is the area of the triangle multiplied?

Ellipse (Conic)

5. A trampoline frame is an ellipse 10 m long and 6 m wide, centred at the origin with its longest measurement along the x -axis.

- a) Write its equation.
- b) Give the coordinates of its vertices and of its foci.

Geometric Transformations in the Cartesian Plane

6. For each mapping rule, name the geometric transformation completely (including its centre, axis or arrow as appropriate) and state whether it is an isometry.

- a) $(x, y) \mapsto (x - 3, y + 5)$
- b) $(x, y) \mapsto (-x, y)$
- c) $(x, y) \mapsto (3x, 3y)$
- d) $(x, y) \mapsto (y, -x)$

Hyperbola (Conic)

7. Consider $\frac{(x-1)^2}{9} - \frac{(y+2)^2}{16} = 1$. Give
- a) its centre and its vertices;
 - b) its foci;
 - c) the equations of its asymptotes.

Parabola (Conic)

8. A parabola has its vertex at the origin and its focus at $(0, 3)$.
- a) Write its equation and the equation of its directrix.
 - b) How wide is the parabola along the horizontal line through the focus?

Points of Intersection Between a Line and a Conic

9. Find the coordinates of the points where the line $y = x + 1$ meets the circle $x^2 + y^2 = 25$.

Points of Intersection Between a Parabola and a Conic

- 10.** Find all points of intersection of the parabola $y = x^2 - 2$ and the circle $x^2 + y^2 = 4$.

Reflections on a Cartesian Plane

- 11.** Triangle ABC has vertices $A(-3, 1)$, $B(2, 4)$ and $C(5, -2)$. Give the coordinates of its image under
- a) the reflection in the x -axis;
 - b) the reflection in the line $y = x$.

Rotations on a Cartesian Plane

- 12.** Give the image of the point $A(5, -2)$ under each rotation centred at the origin.
- a) $r_{(O, 90^\circ)}$
 - b) $r_{(O, 180^\circ)}$
 - c) $r_{(O, -90^\circ)}$

The Composition of Geometric Transformations in a Cartesian Plane

13. Let s be the reflection in the x -axis and t the translation $t_{(3,-2)}$.

- Give the mapping rule of the composition $t \circ s$ (first s , then t).
- Find the image of $A(-2, 5)$ under $t \circ s$.
- Is $t \circ s$ an isometry? Justify.

The Division Point and Midpoint of a Segment

14. A hiking trail runs in a straight line from the trailhead $A(-6, 4)$ to the lookout $B(4, -6)$ (units in hundreds of metres).

- Find the midpoint M of $\overline{AB}$.
- Find the point P that divides $\overline{AB}$ in the ratio $2 : 3$ starting from A .

The Equations of Parallel or Perpendicular Lines

15. Consider the line $d_1 : 2x + 5y = 20$.

- Find the equation of the line perpendicular to d_1 passing through $(6, -1)$.
- Find the equation of the line parallel to d_1 passing through $(0, 3)$.

Translations on a Cartesian Plane

16. A design is moved by the translation $t_{(-5,3)}$.

a) Find the image of the point $A(7, -2)$.

b) A circular hole in the design has equation $(x - 2)^2 + (y + 1)^2 = 9$. Give the equation of its image, and state the image's centre and radius.

Want the harder problems and the worked solutions?

The *Challenge Problems* set and the *Answer Keys* for Analytic Geometry — with the reasoning written out in full — are available at tutorinmontreal.ca.