

Arithmetic

Name: _____

Date: _____

Concepts

- Absolute Value Notation - Secondary 4 and 5
- Arithmetic (theory)
- Logarithms
- Operations (theory)
- Rationalizing Fractions
- Representing Number Sets - Secondary 4 and 5
- The Laws of Exponents
- The Laws of Logarithms
- Using the Laws of Logarithms
- Writing Numbers (theory)

Absolute Value Notation — Secondary 4 and 5

1. At a municipal water plant, a batch of treated water is accepted only if its pH reading p differs from the target value 7.2 by at most 0.4.
- a) Write this acceptance condition using absolute value notation.
 - b) Give the set of acceptable readings in interval notation.
 - c) A batch reads $p = 6.75$. Is it accepted? Support your answer with a calculation.

Arithmetic

2. Each expression below can be written in a simpler *exact* form using one main arithmetic tool from this topic (laws of exponents, laws of logarithms, rationalizing a denominator, absolute value). For each one, name the tool you would use and give the simplified exact value.

a) $\frac{6}{\sqrt{5} - \sqrt{2}}$

b) $\log_3 45 - \log_3 5$

c) $\left(8^{\frac{2}{3}}\right)^{-2}$

d) $|-7| - |3 - 11|$

Logarithms

3. Evaluate without a calculator, showing the exponential form you used in each case.

a) $\log_2 \frac{1}{32}$

b) $\log_9 27$

c) $\log_{0.5} 8$

Operations

4. Evaluate, respecting the priority of operations:

$$E = -2^4 + (-2)^4 - \sqrt[3]{-27} \times |-5 + 2|.$$

Then explain, in one sentence, why -2^4 and $(-2)^4$ do not have the same value.

Rationalizing Fractions

5. Write each quotient with a rational denominator, in simplest exact form.

a) $\frac{10}{\sqrt{8}}$

b) $\frac{3}{2 + \sqrt{7}}$

Representing Number Sets — Secondary 4 and 5

6. A ferry accepts vehicles whose mass is at least 1.5 t and less than 8 t.

a) Write the set of admissible masses (in tonnes) in interval notation and in set-builder notation.

b) List, in extension, the elements of $\mathbb{Z} \cap]-3, 4]$.

The Laws of Exponents

7. Simplify, giving your answers with positive exponents only.

a) $\frac{(2a^3b^{-2})^4}{8a^5b^{-3}}$

b) $\left(\frac{27}{64}\right)^{-\frac{2}{3}}$

The Laws of Logarithms

8. Write each expression as a single logarithm, then evaluate it exactly.

a) $\log_2 40 + \log_2 6 - \log_2 15$

b) $\frac{1}{2}\log_7 81 + \log_7 \frac{7}{9}$

Using the Laws of Logarithms

9. Solve for x , giving the answer to the nearest hundredth:

$$5^{2x-1} = 40.$$

Writing Numbers

10. Write each number in scientific notation, then order the three numbers from smallest to largest. Justify the ordering using the exponents rather than decimal expansions.

a) 0.00042×10^5

b) $\frac{6 \times 10^{-3}}{2 \times 10^4}$

c) $(3 \times 10^2)^{-2}$

Synthesis — drawing on several sheets in this topic

11. The sound level of a machine is $L = 10 \log_{10} \left(\frac{I}{I_0} \right)$ decibels, where I is its intensity in W/m^2 and $I_0 = 10^{-12} \text{ W/m}^2$.

In a workshop, a planer alone registers 78 dB and a dust extractor alone registers 84 dB. When two machines run together their *intensities* add.

- a) Determine the sound level of the workshop when both machines run, to the nearest tenth of a decibel.
- b) A trainee expects $78 + 84 = 162$ dB. Explain, using the laws of logarithms, why that reasoning is wrong.

Want the harder problems and the worked solutions?

The *Challenge Problems* set and the *Answer Keys* for Arithmetic — with the reasoning written out in full — are available at tutorinmontreal.ca.