

Graphs

Name: _____

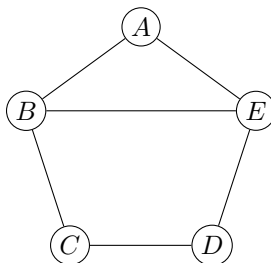
Date: _____

Concepts

- Chains and Cycles
- Graphs
- The Chromatic Number
- The Path of Minimum Value
- The Tree of Minimum or Maximum Values
- Types of Graphs

Chains and Cycles

1. A community garden has five plots, A , B , C , D and E . A gravel path joins two plots exactly when they are drawn joined below.



- Give the degree of each of the five vertices, then state the number of edges of the graph.
- Is $A-B-C-D-E-A$ a cycle? Is it a simple cycle? Justify.
- Find the distance $d(A, C)$ and give a chain that realises it.

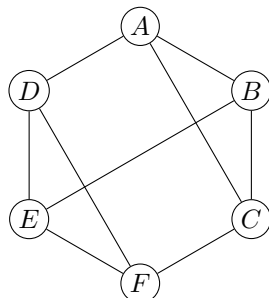
Graphs

2. Six students of a robotics club recorded who they worked with during the term: Amina worked with Bruno, Clara and Dev; Bruno also worked with Clara; Clara also worked with Elin; Dev also worked with Elin; and Elin also worked with Farah. No other pair worked together.

- a) Model the situation with a graph. Say clearly what the vertices and the edges represent, then draw it.
- b) Give the degree of each vertex.
- c) Without recounting the edges one by one, use the degrees to find the number of edges. Check your answer against your drawing.

The Chromatic Number

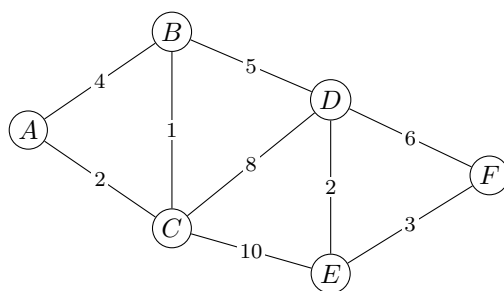
3. Six radio antennas A, B, C, D, E, F interfere with one another. Two antennas joined by an edge below must be given *different* broadcasting frequencies.



- Explain why at least 3 frequencies are needed.
- Give a colouring of the vertices with 3 colours, and state the chromatic number of the graph.

The Path of Minimum Value

4. The graph below shows the walking times, in minutes, between the six pavilions of a science centre.



Find the path of minimum value from A to F and give its value.

The Tree of Minimum or Maximum Values

5. A campus wants to link six buildings by underground fibre. The possible trenches and their costs, in thousands of dollars, are:

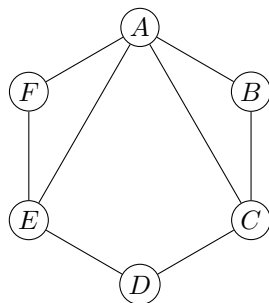
Trench	$A-B$	$A-C$	$B-C$	$B-D$	$C-D$	$C-E$	$D-E$	$E-F$
Cost	7	5	6	9	4	11	3	8

One more trench is possible: $D-F$, at a cost of 12.

- Find a tree of minimum value connecting all six buildings. List its trenches and give the total cost.
- How many trenches does any such tree contain? Explain.
- Find the tree of maximum value and give its total cost.

Types of Graphs

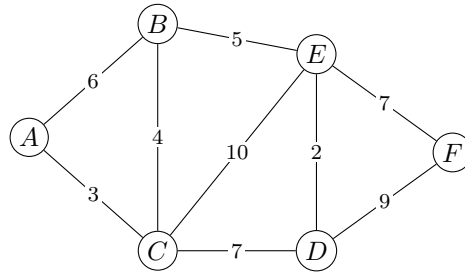
6. The corridors of a museum are modelled by the graph below; the vertices A to F are the rooms.



- a) Is this graph connected? Is it a simple graph? Justify each answer.
- b) Is it a complete graph? Is it a tree? Justify each answer with a count of edges.

Synthesis — drawing on several sheets in this topic

7. A ski resort has six lodges. The graph gives the length, in kilometres, of each existing trail.



- Grooming costs the same per kilometre on every trail. The resort will groom only enough trails to keep all six lodges connected to one another. Which trails should it groom, and what is the total length groomed?
- A patroller must reach lodge F from lodge A as fast as possible. Which route should she take, and how long is it?
- Compare your two answers: does the network chosen in a) contain the route found in b)? What does this show?

Want the harder problems and the worked solutions?

The *Challenge Problems* set and the *Answer Keys* for Graphs — with the reasoning written out in full — are available at tutorinmontreal.ca.