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Functions for Business Models

Name: ______________________________ Date: ______________

Concepts

- Linear Models for Cost, Revenue, Supply and Demand
- Quadratic Functions and Maximum Revenue (theory)
- Exponential Functions and Growth Models (theory)
- Logarithmic Functions and the Laws of Logarithms
- Solving Exponential and Logarithmic Equations

Linear Models for Cost, Revenue, Supply and Demand

1. A print shop makes custom notebooks. Its fixed costs are \$1 800 a month and each notebook costs \$4.50 in paper, binding and labour. Every notebook sells for \$12. Let x be the number of notebooks made and sold in a month.

a) Write the cost $C(x)$, the revenue $R(x)$ and the profit $P(x)$, in dollars.

b) Find the break-even quantity.

c) What is the profit or loss in a month when 150 notebooks are sold?

d) How many notebooks must be sold in a month for a profit of \$1 200?

2. A wholesaler sells insulated water bottles. Market research gives two observations of weekly demand and two of weekly supply, where p is the unit price in dollars and x the number of bottles. At \$30, buyers demand 900 bottles and suppliers offer 600; at \$40, buyers demand 700 bottles and suppliers offer 1 000. Assume both relations are linear.

a) Find the price-demand function $p = D(x)$ and the price-supply function $p = S(x)$.

b) Find the equilibrium quantity and the equilibrium price.

c) At a price of \$40, is there a shortage or a surplus, and of how many bottles?

Quadratic Functions and Maximum Revenue

3. A small workshop makes leather phone cases. Its price-demand function is

$$p = D(x) = 90 - \tfrac{1}{50}x, \qquad 0 \le x \le 4500,$$

where x is the number of cases sold a month and p the price in dollars.

a) Write the revenue function $R(x)$.

b) Find the quantity and the price that maximize revenue, and the maximum revenue.

c) Find the monthly revenue at a price of \$60, and the one other price that brings in exactly the same revenue.

Exponential Functions and Growth Models

4. A consultancy buys a laptop for \$2 400. Its book value falls by the same percentage every year, and after 2 years it is worth \$1 350.

a) Find the annual rate of depreciation and write the value $V(t)$ after t years.

b) Find the value after 3 years.

c) Write the same model in the form $V(t) = 2400e^{-kt}$, giving k exactly.

d) After how many full years is the book value first below \$500?

Logarithmic Functions and the Laws of Logarithms

5.

a) Write $\ln\left(\dfrac{x^3\sqrt{y}}{z^2}\right)$, for $x, y, z > 0$, as a sum and difference of multiples of $\ln x$, $\ln y$ and $\ln z$.

b) Write $3\ln a - \frac{1}{2}\ln b + \ln 4$, for $a, b > 0$, as a single logarithm.

c) Given $\ln 2 = u$, $\ln 3 = v$ and $\ln 5 = w$, express $\ln 360$, $\ln 0.15$ and $\ln\sqrt{1.2}$ in terms of u, v and w.

6. A marketing firm models a client's weekly sales S, in thousands of dollars, as a function of the weekly advertising budget x, in thousands of dollars:

$$S(x) = 30 + 8\ln x, \qquad x \geq 1.$$

a) Find the weekly sales on a budget of \$1 000.

b) Use the laws of logarithms to show that doubling *any* budget raises weekly sales by the same amount, and give that amount exactly.

c) Find, exactly and then to the nearest dollar, the increase in weekly sales when the budget rises from \$5 000 to \$15 000. Use $\ln 3 \approx 1.0986$.

Solving Exponential and Logarithmic Equations

7. Solve each equation exactly. Check every candidate against the domain of the original equation.

a) $5e^{0.2t} = 40$

b) $3^{2x-1} = 7^x$

c) $\ln(x+2) + \ln(x-1) = \ln 10$

8. A streaming start-up has 2 000 subscribers and an established rival has 6 000. Their subscriber counts t months from now are modelled by

$$N_A(t) = 2000e^{0.3t} \quad \text{(start-up)}, \qquad N_B(t) = 6000e^{0.1t} \quad \text{(rival)}.$$

When will the two services have the same number of subscribers, and how many will each have then? Give exact answers, then decimals, using $\ln 3 \approx 1.0986$ and $\sqrt{3} \approx 1.7321$.

Synthesis — drawing on several topics in this unit

9. A phone-accessory maker launches a charging stand. Weekly sales t weeks after launch are expected to follow $q(t) = 3000e^{-0.1t}$ units. Each stand sells for \$15 and costs \$5 to make, and the product line carries fixed costs of \$7 500 a week.

a) Write the weekly profit $P(t)$ and find the profit in the launch week ($t = 0$).

b) Find, exactly and to two decimal places, the time at which weekly profit falls to zero. Use $\ln 2 \approx 0.6931$.