

Mathematics of Finance

Name: _____

Date: _____

Concepts

- Arithmetic and Geometric Sequences (theory)
- Simple Interest and Maturity Value
- Compound Interest and Future Value
- Present Value and the Effective Annual Rate
- Continuous Compounding and Solving for Time or Rate (theory)
- Future Value of an Annuity and Sinking Funds
- Present Value of an Annuity and Loan Amortization

Arithmetic and Geometric Sequences

1.

- a) A courier company leases a warehouse. The rent is \$24 000 in the first year and rises by \$600 every year after that. Find the rent in year 10 and the total rent paid over the first 10 years.
- b) A start-up spends \$500 on online advertising in its first month and raises the budget by 20% every month after that. Find the budget in month 6 and the total spent over the first 6 months. Use $1.2^5 = 2.48832$ and $1.2^6 = 2.985984$.

Simple Interest and Maturity Value

2. A catering company borrows \$8 400 on 3 March 2025 at 6.5% simple interest per year and repays principal and interest in a single payment on 31 July 2025. Interest is charged for the exact number of days, counting the day the money is borrowed but not the day it is repaid, on a 365-day year. (2025 is not a leap year.)

- a) Find the number of days in the term of the loan.
- b) Find the interest and the maturity value.

3. A treasury bill pays no interest: it is bought below its face value and redeemed for its face value on its maturity date. Its return is quoted as a simple annual interest rate earned on the price paid, on a 365-day year.

- a) A 182-day treasury bill with a face value of \$25 000 is bought for \$24 500. Find its yield, as a percentage to two decimal places.
- b) What price should an investor pay for the same bill to earn a yield of exactly 4%? Round to the cent.

Compound Interest and Future Value

4. A physiotherapy clinic places \$12 000 in a 5-year guaranteed investment certificate paying 3.6% per year, compounded monthly. No money is added or withdrawn. Find the value at maturity and the total interest earned. Use $(1.003)^{60} = 1.196\,894\,803$.

5. A tutoring agency deposits \$15 000 in an account paying 4.8% per year, compounded quarterly, and makes no further deposits or withdrawals.

a) Find the balance after 2 years and after 3 years.

b) How much interest does the account earn during the third year alone? Explain why it is more than the interest earned during the first year.

Use $(1.012)^4 = 1.048\,870\,933$, $(1.012)^8 = 1.100\,130\,234$ and $(1.012)^{12} = 1.153\,894\,624$.

Present Value and the Effective Annual Rate

6. A food-truck owner is offered \$40 000 for her truck, to be paid in a single amount 3 years from now. Money can earn 4.8% per year, compounded monthly. What is the offer worth today, and how much less is that than its face amount? Use $(1.004)^{-36} = 0.866\,136\,496$.

7. A store credit card charges 18% per year, compounded monthly.

a) Find the effective annual rate, as a percentage to two decimal places.

b) Find the nominal annual rate compounded quarterly that is equivalent to it, as a percentage to two decimal places.

Use $(1.015)^3 = 1.045\,678\,375$ and $(1.015)^{12} = 1.195\,618\,171$.

Continuous Compounding and Solving for Time or Rate

8. A ski-rental shop invests \$6 000 at 4.5% per year, compounded continuously.

a) Find the value after 8 years.

b) How long does it take for the investment to reach \$9 000? Give the exact time and the time to two decimal places.

Use $e^{0.36} = 1.433\,329\,415$ and $\ln 1.5 = 0.405\,465$.

Future Value of an Annuity and Sinking Funds

9. A dental clinic deposits \$750 at the end of every month for 5 years into an account paying 4.2% per year, compounded monthly. Find the balance just after the last deposit, and how much of it is interest.

Use $(1.0035)^{60} = 1.233\,225\,821$.

10. A ski-lodge operator must replace its snowmobile fleet in 6 years at an estimated cost of \$120 000. It sets up a sinking fund with equal deposits at the end of every quarter, in an account paying 5% per year, compounded quarterly. Use $(1.0125)^{24} = 1.347\,351\,050$.

- a) Find the quarterly deposit.
- b) Find the total deposited and the total interest earned.
- c) Build the first two rows of the sinking-fund schedule: the interest earned and the balance just after each of the first two deposits.

Present Value of an Annuity and Loan Amortization

11. A landscaping company finances a \$28 000 pickup truck with a 5-year loan at 6% per year, compounded monthly, repaid by equal payments at the end of every month. Find the monthly payment and the total interest paid over the life of the loan. Use $(1.005)^{-60} = 0.741\,372\,196$.

12. A café borrows \$15 000 at 8% per year, compounded annually, and repays it with four equal payments at the end of each year.

a) Find the annual payment.

b) Construct the amortization schedule: for each year, the payment, the interest, the principal repaid and the balance after the payment. Round each interest amount to the cent, and adjust the last payment so that the balance ends at exactly zero.

Use $(1.08)^{-4} = 0.735\,029\,853$.

Synthesis — drawing on several topics in this unit

13. A water-taxi operator saves for a down payment by depositing \$3 000 at the end of every quarter for 3 years in an account paying 4% per year, compounded quarterly. Then it uses the whole balance as a down payment on a \$180 000 boat and finances the rest with a 10-year loan at 6% per year, compounded monthly, repaid by equal payments at the end of every month.

a) Find the down payment.

b) Find the amount financed and the monthly loan payment.

Use $(1.01)^{12} = 1.126\,825\,030$ and $(1.005)^{-120} = 0.549\,632\,733$.

Want the harder problems and the worked solutions?

The *Challenge Problems* set and the *Answer Keys* for Mathematics of Finance — with the reasoning written out in full — are available at tutorinmontreal.ca.