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Linear Inequalities and Linear Programming

Name: ________________________________ Date: ______________

Concepts

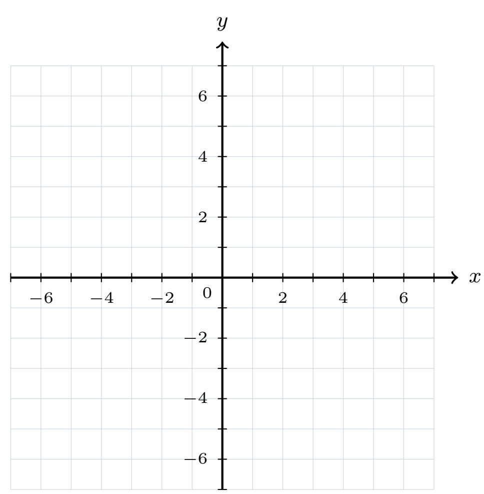
- Graphing a Linear Inequality in Two Variables (theory)
- Systems of Linear Inequalities and the Feasible Region
- Corner Points of Bounded and Unbounded Regions (theory)
- Formulating a Linear Programming Model
- Solving a Linear Program Graphically

Graphing a Linear Inequality in Two Variables

1. A pottery studio fires x batches of mugs and y batches of bowls in its kiln each week. A batch of mugs needs 3 hours of kiln time and a batch of bowls 2 hours, and the kiln is available for at most 12 hours a week.

a) Write the kiln-time constraint as an inequality. Graph its solution set in the whole plane on the grid: give the intercepts of the boundary line, say whether the line is solid or dashed, and state the test point you used to choose the side.

b) The studio can only fire whole batches, and neither x nor y can be negative. Describe the part of your half-plane that the studio can actually use, and count the possible weekly plans (x, y), including the plan of firing nothing.

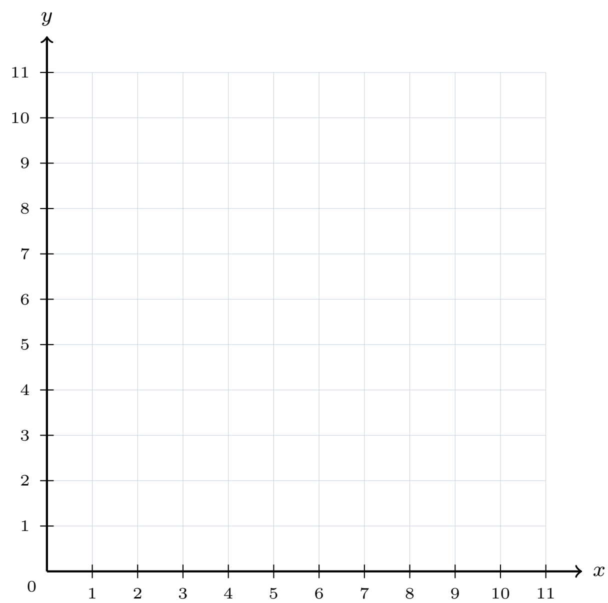


Systems of Linear Inequalities and the Feasible Region

2. A soap maker makes x batches of bar soap and y batches of liquid soap a week. A batch of bar soap needs 1 hour of mixing and 3 curing racks; a batch of liquid soap needs 2 hours of mixing and 2 curing racks. There are 10 mixing hours and 18 curing racks available each week.

a) Write the system of inequalities, including the non-negativity constraints, and shade its feasible region on the grid.

b) For each of the plans $(5, 2)$, $(2, 4)$, $(4, 3)$ and $(1, 5)$, say whether it is feasible; for each one that is not, name the constraint it breaks.

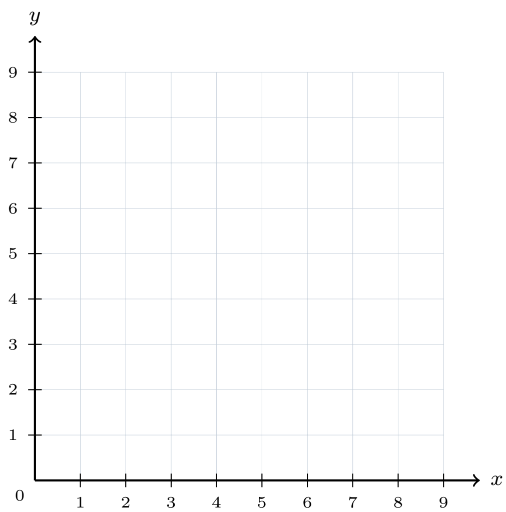


3. A florist has a contract to deliver at least 8 arrangements on a Saturday: x small ones and y large ones. A small arrangement takes 2 units of space in her van and a large one 3 units, and the van holds 12 units.

a) Write the system of inequalities, including non-negativity.

b) Graph it on the grid and show that its feasible region is empty. What does that mean for the florist?

c) She can rent a larger van. Find the least van space, in units, that makes at least one delivery plan possible, and the plan that uses it.



Corner Points of Bounded and Unbounded Regions

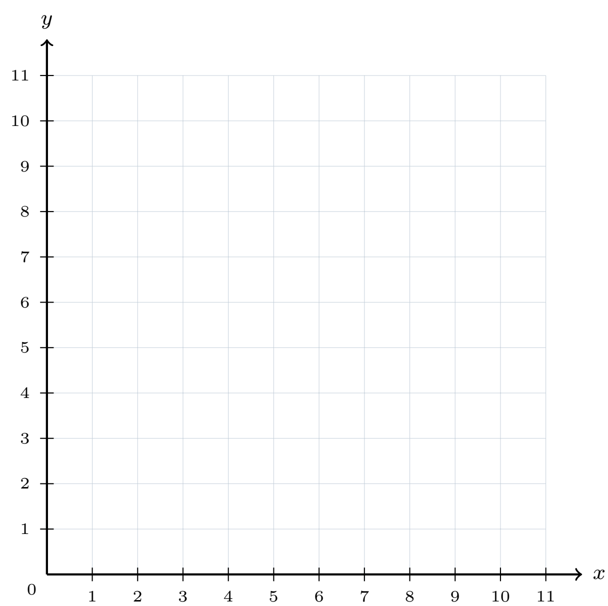
4. Consider the system

$$x + y \geq 6, \qquad x + 3y \geq 10, \qquad 3x + y \geq 10, \qquad x \geq 0, \qquad y \geq 0.$$

a) Graph the feasible region on the grid and find all of its corner points, solving for each intersection algebraically.

b) Is the region bounded or unbounded? Explain.

c) The boundary lines $x + 3y = 10$ and $3x + y = 10$ meet at $\left(\frac{5}{2}, \frac{5}{2}\right)$. Explain why this point is not a corner point of the region.



Formulating a Linear Programming Model

5. A skate shop assembles x street decks and y longboards a week. A street deck needs 1 hour of assembly and 2 hours of finishing; a longboard needs 2 hours of assembly and 1 hour of finishing. Each week there are 40 hours of assembly and 50 hours of finishing available. At most 18 longboards can be sold a week, and a school club has a standing order for at least 5 street decks a week. The profit is $45 on a street deck and $60 on a longboard.

a) Define the decision variables with their units and write the linear programming model: the objective and every constraint. Do not solve it.

b) Is the plan of 14 street decks and 12 longboards feasible? If it is, find its profit and the hours of assembly and of finishing it leaves unused.

6. A feed mill blends x tonnes of grain with y tonnes of soy meal each day. Grain is 10% protein and costs $300 a tonne; soy meal is 40% protein and costs $600 a tonne. The mill must produce at least 50 tonnes of feed a day, the feed must be at least 16% protein by weight, and at most 30 tonnes of soy meal can be delivered a day. The mill wants the cheapest blend.

a) Write the protein requirement as an inequality, and simplify it to the form $ax + by \leq 0$ with integer coefficients.

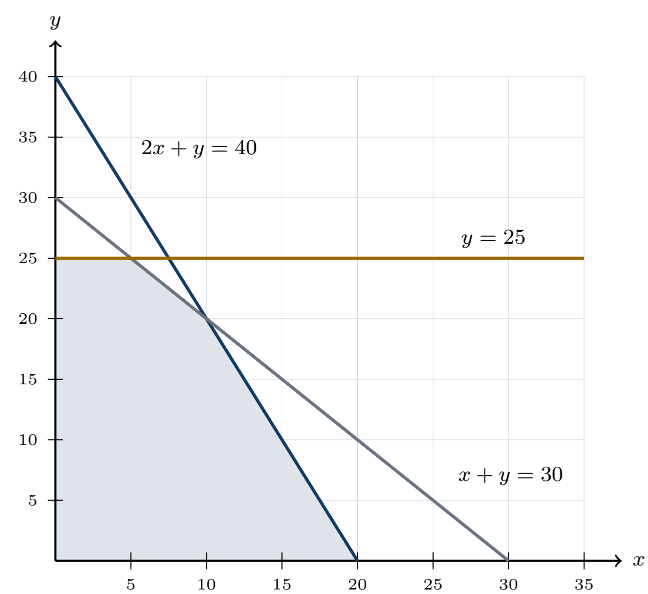
b) Write the complete linear programming model. Do not solve it.

Solving a Linear Program Graphically

7. A cider mill makes x cases of still cider and y cases of sparkling cider a week. A case of still cider needs 2 hours of pressing and 1 hour of bottling; a case of sparkling needs 1 hour of each. There are 40 pressing hours and 30 bottling hours a week, and at most 25 cases of sparkling cider can be sold. The profit is \$40 on a case of still cider and \$50 on a case of sparkling. The model is

$$\text{Maximize } P = 40x + 50y \quad \text{subject to} \quad 2x + y \leq 40, \quad x + y \leq 30, \quad y \leq 25, \quad x, y \geq 0,$$

and its feasible region is shaded below (one grid square is 5 cases each way).



a) Find every corner point of the region, solving algebraically for any that is not on an axis.

b) Find the production plan that maximizes profit, and the maximum profit.

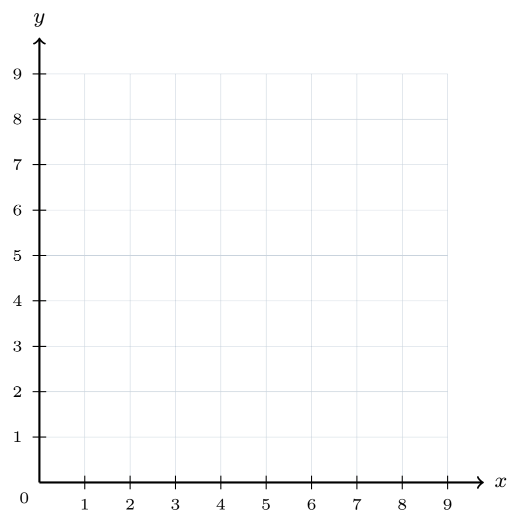
c) At that plan, which constraints are used up exactly, and how much of the other resource is left over?

8. A farm co-operative buys two fertilizers for a field. A bag of brand A holds 2 kg of nitrogen and 1 kg of phosphate and costs \$25; a bag of brand B holds 1 kg of nitrogen and 2 kg of phosphate and costs \$20. The field needs at least 8 kg of nitrogen and at least 7 kg of phosphate. Let x and y be the numbers of bags of A and B.

a) Write the model that minimizes the cost, graph its feasible region on the grid, and find its corner points.

b) Find the cheapest purchase and its cost.

c) Explain why the cost has a minimum on this region but no maximum.



Synthesis — drawing on several topics in this unit

9. A hot-sauce company makes x cases of mild sauce and y cases of fiery sauce a week. A case of mild uses 2 kg of peppers and 40 minutes on the bottling line; a case of fiery uses 4 kg of peppers and 20 minutes. Each week 100 kg of peppers and 1 100 minutes of bottling are available, and a restaurant chain has ordered at least 5 cases of fiery. The profit is \$30 on a case of mild and \$50 on a case of fiery.

a) Formulate the linear programming model.

b) Graph the feasible region on the grid below (one square is 5 cases each way) and find its corner points.

c) Find the weekly plan that maximizes profit, and the maximum profit.