

Counting and Probability

Name: _____

Date: _____

Concepts

- The Addition and Multiplication Principles (theory)
 - Sample Spaces, Events and Equally Likely Outcomes (theory)
 - Permutations and Factorial Notation
 - Union, Intersection and Complement of Events
 - Combinations and Mixed Counting Problems
 - Conditional Probability, Probability Trees and Independence
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The Addition and Multiplication Principles

1.

- a) A café's lunch combo is a main, a drink and an optional dessert. The main is one of 5 sandwiches or one of 3 salads; the drink is one of 4; the dessert is one of 2 pastries, or none. How many different combos are there?
- b) A hardware store labels each product with a code made of one letter (26 choices) followed by three digits, where the first digit may not be 0. How many codes are possible?
- c) How many of the codes in (b) have three different digits?

Permutations and Factorial Notation**2.**

- a) Evaluate $\frac{10!}{7!}$ and simplify $\frac{(n+1)!}{(n-1)!}$ for an integer $n \geq 1$.
- b) Solve $\frac{n!}{(n-2)!} = 56$ for the integer $n \geq 2$.
- c) A sales representative must visit 6 client offices tomorrow, one after another. In how many orders can she make the visits?
- d) The 9 members of a co-operative's board elect a chair, a vice-chair and a treasurer, three different people. In how many ways can the three posts be filled?

3. A bookstore sets out 7 different books in a row in its window: 3 business titles and 4 novels.

- a) In how many arrangements are all 3 business titles side by side, in one unbroken group?
- b) In how many arrangements do novels and business titles alternate?
- c) The shop next door lines up 9 mugs on a shelf: 4 identical red ones, 3 identical blue ones and 2 identical white ones. How many different-looking rows are possible?

Combinations and Mixed Counting Problems

4. A firm forms a project team of 4 from 7 accountants and 5 analysts.

- a) How many teams are possible?
- b) How many teams have exactly 2 accountants and 2 analysts?
- c) How many teams include at least one analyst?

5.

- a) A pizzeria offers 12 toppings, 3 sizes and 2 crusts. How many different pizzas with exactly 3 different toppings can be ordered?
- b) A coffee chain will open 3 new branches, chosen from 8 possible sites. In how many ways can the 3 sites be chosen? In how many ways if the chain also decides the order in which the three will open?
- c) In how many ways can the 3 sites be chosen if one particular site, the downtown one, must be among them?

Sample Spaces, Events and Equally Likely Outcomes

6. A manager will invite 2 of 5 job applicants — Ana, Ben, Chloé, Dev and Emma — to a second interview, choosing the pair at random so that every pair is equally likely.

- a) List the sample space.
- b) Find the probability that Ana is invited.
- c) Find the probability that neither Ben nor Chloé is invited.
- d) Find the probability that Ana or Ben (or both) is invited.

Union, Intersection and Complement of Events

7. At a wholesaler, an invoice is chosen at random. Let L be the event that it was paid late and E the event that it contains a billing error. The records give $P(L) = 0.35$, $P(E) = 0.20$ and $P(L \cap E) = 0.12$. Find the probability that the invoice

- a) was paid late or contains an error;
- b) was paid on time and contains no error;
- c) was paid late but contains no error;
- d) has exactly one of the two problems.

8. An office has 10 laptops, 3 of which have a faulty battery. A technician takes 4 of them at random for a training session. Find the probability that
- a) none of the 4 has a faulty battery;
 - b) at least one has a faulty battery;
 - c) exactly one has a faulty battery;
 - d) at least two have a faulty battery.

Conditional Probability, Probability Trees and Independence

9. A hotel keeps 12 sealed gift cards in a drawer for a staff draw: 4 are worth \$50 and 8 are worth \$20. The manager draws 2 cards at random, one after the other, without replacement.
- a) Draw a probability tree for the two draws, with the probability on every branch.
 - b) Find the probability that the two cards are worth \$70 in total.
 - c) Find the probability that at least one card is worth \$50.
 - d) Given that the first card is worth \$20, what is the probability that the second is worth \$50?

10. For a loan application chosen at random at a credit union, let A be the event that it is approved and S the event that the applicant is self-employed. Records give $P(A) = 0.6$, $P(S) = 0.25$ and $P(A \cap S) = 0.1$.

- a) Find $P(A | S)$ and $P(S | A)$, and say in words what each means.
- b) Find the approval rate for applicants who are not self-employed.
- c) Are the events A and S independent? Justify.

Synthesis — drawing on several topics in this unit

11. Six sales representatives, including Lina and Omar, are given the six presentation slots at a trade show (slots 1 to 6, in order), with every assignment equally likely.

- a) Find the probability that Lina presents before Omar.
- b) Find the probability that Lina and Omar present in consecutive slots, in either order.
- c) Given that Lina has slot 1, find the probability that Omar has slot 6.
- d) Given that Lina presents before Omar, find the probability that Lina has slot 1.

Want the harder problems and the worked solutions?

The *Challenge Problems* set and the *Answer Keys* for Counting and Probability — with the reasoning written out in full — are available at tutorinmontreal.ca.