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Limits, Derivatives and Marginal Analysis

Name: ____ Date: __________________

Concepts

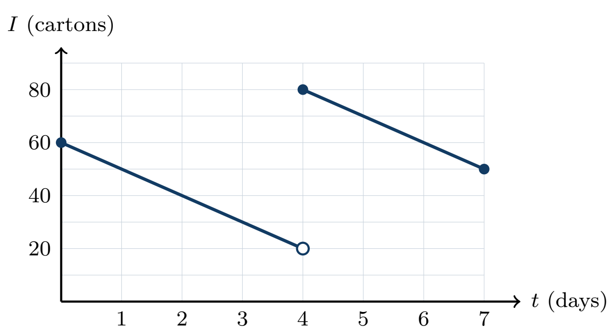
- Limits from Graphs and by Algebra
- Infinite Limits, Limits at Infinity and Asymptotes
- Continuity and Sign Charts (theory)
- Rates of Change and the Definition of the Derivative
- Power, Sum and Constant Multiple Rules
- Marginal Cost, Revenue and Profit

Limits from Graphs and by Algebra

1. A stationery shop tracks its stock of printer-paper cartons. Stock falls by 10 cartons a day; a delivery arrives at the start of day 4. The stock $I(t)$, in cartons, t days after the count begins, is

$$I(t) = \begin{cases} 60 - 10t, & 0 \le t < 4, \\ 120 - 10t, & 4 \le t \le 7, \end{cases}$$

and its graph is shown: a segment from $(0, 60)$ falling to an open circle at $(4, 20)$, then a solid dot at $(4, 80)$ and a segment falling from it to $(7, 50)$.



a) Find $\lim_{t \to 2} I(t)$.

b) Find $\lim_{t \to 4^-} I(t)$, $\lim_{t \to 4^+} I(t)$ and $I(4)$.

c) Does $\lim_{t \to 4} I(t)$ exist? Justify.

d) In one sentence, what does $\lim_{t \to 4^-} I(t)$ tell the shop manager?

2. Evaluate each limit exactly. Try direct substitution first, and show the algebra that removes a $\frac{0}{0}$ form wherever one appears.

a) $\lim_{x\to4} \dfrac{x^2-2x-8}{x^2-16}$

b) $\lim_{x\to5} \dfrac{\sqrt{x+4}-3}{x-5}$

c) $\lim_{x\to2} \dfrac{\dfrac{1}{x}-\dfrac{1}{2}}{x-2}$

d) $\lim_{x\to3} \dfrac{x^2+1}{x+2}$

Infinite Limits, Limits at Infinity and Asymptotes

3. A board-game publisher has fixed costs of \$18 000 for a print run, plus \$9 for each copy printed. Let $x > 0$ be the number of copies.

a) Write the average cost per copy, $\bar{C}(x) = \dfrac{C(x)}{x}$, and find $\bar{C}(1\,000)$ and $\bar{C}(6\,000)$.

b) Find $\lim\limits_{x\to\infty} \bar{C}(x)$ and $\lim\limits_{x\to 0^+} \bar{C}(x)$, and give the equations of the horizontal and vertical asymptotes of the graph of $\bar{C}$.

c) Explain in one sentence why no print run, however large, brings the average cost down to \$9.00 a copy, and find the run that brings it to \$10.00.

4. Let

$$g(x) = \frac{2x^2 - 18}{x^2 + 2x - 15}.$$

a) The denominator is zero at two values of x. For each, decide whether the graph of g has a vertical asymptote or a hole there, and justify with a limit.

b) At the vertical asymptote, find both one-sided limits.

c) Find $\lim_{x\to\infty} g(x)$ and $\lim_{x\to-\infty} g(x)$, and state the horizontal asymptote.

d) For $h(t) = \dfrac{900}{3 + 2e^{-0.5t}}$, find $\lim_{t\to\infty} h(t)$ and $\lim_{t\to-\infty} h(t)$, and state both horizontal asymptotes of its graph.

Continuity and Sign Charts

5. Let

$$f(x) = \frac{x^2 - 2x - 8}{x - 1}.$$

a) On which intervals is f continuous?

b) Build a sign chart for f, and use it to solve $f(x) \leq 0$. Give the answer in interval notation.

Rates of Change and the Definition of the Derivative

6. A coffee roaster's weekly cost of roasting x kilograms of beans is

$$C(x) = 200 + 6x + 0.01x^2 \text{ dollars.}$$

a) Find the average rate of change of cost as production rises from 100 kg to 150 kg, with units.

b) Simplify $\dfrac{C(100 + h) - C(100)}{h}$ for $h \neq 0$.

c) Let $h \to 0$ to find the instantaneous rate of change of cost at 100 kg, and say in one sentence what it means.

7. A preserves maker finds that it can sell x hundred jars of jam a month at a price of

$$p = D(x) = \frac{180}{x+3} \text{ dollars per jar,} \qquad x \geq 0.$$

a) Use the definition $D'(x) = \lim_{h \to 0} \frac{D(x+h) - D(x)}{h}$ to find $D'(x)$.

b) Find $D(3)$ and $D'(3)$, and interpret $D'(3)$ with units.

Power, Sum and Constant Multiple Rules

8. Differentiate each function. Rewrite it first as a sum of constant multiples of powers of x where that is needed, and give the answer without negative or fractional exponents in (b).

a) $f(x) = 4x^5 - 3x^2 + 7x - 9$

b) $g(x) = 6\sqrt{x} - \frac{8}{x^2} + \sqrt[3]{x^2}$, for $x > 0$

c) $h(x) = (x^2 + 3)^2$

d) $k(x) = \frac{3x^3 - 2x + 5}{x}$, for $x \neq 0$

9. Let $f(x) = 2x^3 - 3x^2 - 36x + 5$.

a) Find every point on the graph of f at which the tangent line is horizontal.

b) Find the equation of the tangent line at $x = 1$.

Marginal Cost, Revenue and Profit

10. A candle maker's weekly cost of making x candles is

$$C(x) = 800 + 6x - 0.004x^2 \text{ dollars}, \qquad 0 \le x \le 600.$$

a) Find the marginal cost function $C'(x)$.

b) Find $C'(250)$ and interpret it.

c) Find the exact cost of making the 251st candle, and compare it with $C'(250)$.

11. A small workshop sells yoga mats. At a price of p dollars it sells x mats a week, where $p = 120 - 0.04x$ for $0 \leq x \leq 3\,000$. Its weekly cost is $C(x) = 3\,000 + 40x$ dollars.

a) Write the revenue $R(x)$ and the profit $P(x)$, and find the marginal revenue $R'(x)$ and the marginal profit $P'(x)$.

b) Find $R'(500)$, $P'(800)$ and $P'(1\,200)$, and interpret each.

Synthesis — drawing on several topics in this unit

12. A ski-rental shop can rent x ski packages a week at a price of $p = 80 - 0.1x$ dollars each, for $0 \leq x \leq 800$. Its weekly cost is $C(x) = 5\,000 + 20x$ dollars.

a) Write the weekly profit $P(x)$.

b) Use a sign chart to find the numbers of weekly rentals at which the shop makes a profit, that is, at which $P(x) > 0$.

c) Find the marginal profit $P'(x)$, and use its sign to say for which numbers of rentals one more rental raises profit and for which it lowers it.