University Business Mathematics · Practice Worksheet · tutorinmontreal.ca

Curve Analysis and Business Optimization

Name: ____ Date: __________________

Concepts

- The First Derivative, Critical Numbers and Local Extrema
- Concavity, Inflection Points and Diminishing Returns
- Sketching a Curve with Its Asymptotes (theory)
- Absolute Extrema on an Interval
- Maximizing Revenue and Profit
- Minimizing Cost and Average Cost (theory)

The First Derivative, Critical Numbers and Local Extrema

1. For each function, find every critical number, say whether f' is zero or undefined there, and use a sign analysis of the first derivative to classify each one as a local maximum, a local minimum or neither. Give the value of the function at each local extremum.

a) $f(x) = 3x^5 - 20x^3$

b) $g(x) = x - 3x^{1/3}$

2. A tutoring agency opens a new branch. The number of students enrolled t months after opening is modelled by

$$N(t) = t^3 - 18t^2 + 81t + 40, \qquad 0 \leq t \leq 12.$$

a) On which time intervals is enrolment rising, and on which is it falling?

b) Find each local maximum and local minimum of enrolment, with the month at which it occurs, and describe in one sentence what happens to enrolment over the year.

Concavity, Inflection Points and Diminishing Returns

3. For each function, find the intervals on which the graph is concave up and those on which it is concave down, and give the coordinates of every inflection point.

a) $f(x) = \ln(x^2 + 4)$

b) $g(x) = x^4 + 2x^3 - 36x^2$

4. A regional furniture retailer finds that its monthly revenue R, in thousands of dollars, depends on its monthly advertising budget x, in thousands of dollars:

$$R(x) = -x^3 + 36x^2 + 50x + 300, \qquad 0 \le x \le 20.$$

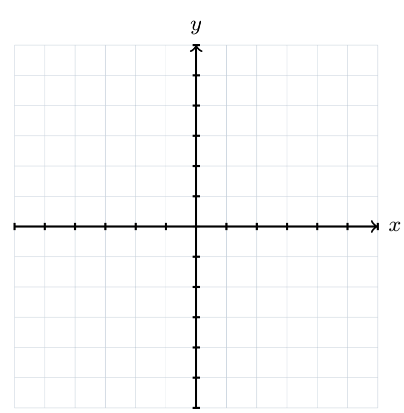
a) Show that revenue increases with the advertising budget over the whole range.

b) Find the point of diminishing returns: the budget at which the rate of increase of revenue is greatest. Give the revenue and the value of R' there, with units.

c) Compute $R'(8)$ and $R'(16)$ and say in one sentence what the comparison tells the retailer.

Sketching a Curve with Its Asymptotes

5. Let $f(x) = \dfrac{4x}{(x-1)^2}$. Find the domain and intercepts; the vertical and horizontal asymptotes, each justified with a limit; the intervals of increase and decrease and any local extremum; the intervals of concavity and any inflection point. Then sketch the curve on the grid, where each square is one unit, drawing each asymptote as a dashed line.



Absolute Extrema on an Interval

6. Find the absolute maximum and the absolute minimum of each function on the given interval, and say where each occurs. Show the list of candidates you compare.

a) $g(x) = 2x^3 - 15x^2 + 36x$ on $[1, 5]$

b) $h(x) = x - 2\ln x$ on $[1, e^2]$. Give exact values, then decimals using $\ln 2 \approx 0.6931$ and $e^2 \approx 7.389$.

7. A ski-rental shop is open from 9 a.m. to 5 p.m. The number of pairs of skis out on rental t hours after opening is modelled by

$$N(t) = 4t^3 - 60t^2 + 225t + 20, \qquad 0 \le t \le 8.$$

Find the largest and the smallest number of pairs out on rental during the day, and the clock time or times at which each occurs.

Maximizing Revenue and Profit

8. An online course platform sells a certification course. At a price of p dollars, it expects $x = 3000e^{-p/40}$ enrolments a month.

a) Write the monthly revenue $R(p)$ as a function of the price.

b) Find the price that maximizes revenue, and justify that it is a maximum.

c) Find the number of enrolments a month at that price, exactly and to the nearest whole enrolment, and the maximum revenue, exactly and to the nearest cent. Use $e^{-1} \approx 0.3678794412$.

9. A print studio sells framed prints. The price-demand function is $p = 170 - 0.5x$ dollars, where x is the number of prints sold a week, and the weekly cost is $C(x) = 4000 + 40x + 0.001x^3$ dollars, for $0 \leq x \leq 340$.

a) Write the weekly profit $P(x)$.

b) Find the number of prints and the price that maximize weekly profit, and the maximum profit. Confirm that it is a maximum.

c) Check that marginal revenue equals marginal cost at that output.

Minimizing Cost and Average Cost

10. A furniture maker's monthly cost of making x chairs is

$$C(x) = 40\,000 + 400x - 3x^2 + 0.01x^3 \text{ dollars}, \qquad x > 0.$$

a) Write the average cost per chair $\bar{C}(x)$.

b) Find the production level that minimizes the average cost, and the minimum average cost. (The equation you reach has exactly one real root, a multiple of 100.)

c) Find the production level at which the marginal cost $C'(x)$ is smallest, and check that at the level found in (b) the marginal cost equals the average cost.

Synthesis — drawing on several topics in this unit

11. A small electronics assembler's weekly profit, in dollars, from x hundred units is

$$P(x) = -x^3 + 75x^2 - 675x - 5000, \qquad 0 \le x \le 60,$$

where 60 hundred units is the plant's weekly capacity.

a) Find the intervals on which profit increases and decreases, and classify each critical number.

b) Find the inflection point and explain what it means for the marginal profit.

c) Find the absolute maximum and minimum profit on $[0, 60]$, and the output at each.