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Integral Calculus for Business

Name: ____ Date: __________________

Concepts

- Antiderivatives and Indefinite Integrals
- Integration by Substitution
- Riemann Sums and the Definite Integral (theory)
- The Fundamental Theorem and Net Change
- Area Between Two Curves
- Consumer and Producer Surplus (theory)
- The Lorenz Curve and the Gini Index (theory)

Antiderivatives and Indefinite Integrals

1. Find each indefinite integral, and state an interval on which your answer is valid. Check part (a) by differentiating.

a) $\int \left(12x^3 - 6\sqrt{x} + 1\right)\,dx$

b) $\int \left(\frac{7}{x} - 3e^x\right)\,dx$

c) $\int \frac{(2x+1)^2}{x^2}\,dx$

2. A food truck sells x meals a day, $0 \leq x \leq 200$. Its marginal cost is $C'(x) = 12 - 0.04x$ dollars per meal and its fixed costs are \$350 a day. Its marginal revenue is $R'(x) = 20 - 0.02x$ dollars per meal.

a) Find the daily cost function $C(x)$.

b) Find the revenue function $R(x)$, saying why its constant of integration is 0, and deduce the price-demand function $p = D(x)$.

c) Find the profit on a day when 100 meals are sold.

Integration by Substitution

3. Evaluate each integral, stating the substitution you use.

a) $\int (x+1)\left(x^2+2x-5\right)^4\,dx$

b) $\int x^2 e^{x^3-1}\,dx$

c) $\int \frac{1}{x\ln x}\,dx$, for $x > 1$

4. A furniture workshop makes x dining chairs a month. Its marginal cost is

$$C'(x) = 150 + \frac{90}{\sqrt{2x+9}} \quad \text{dollars per chair,}$$

and its fixed costs are \$2 000 a month, so $C(0) = 2000$.

a) Find the monthly cost function $C(x)$.

b) Find the total cost of a month in which 20 chairs are made.

Riemann Sums and the Definite Integral

5. A ceramics studio makes x mugs a week. Its marginal cost is

$$C'(x) = 0.01x^2 - x + 40 \quad \text{dollars per mug}, \qquad 0 \le x \le 40,$$

which decreases on this interval. The increase in weekly cost when output goes from 0 to 40 mugs is $\displaystyle\int_0^{40} C'(x)\,dx$.

a) Split $[0, 40]$ into 4 subintervals of equal width. Write the right-endpoint sum R_4 in sigma notation, then evaluate it and the left-endpoint sum L_4.

b) Which of the two overestimates the increase in cost, and why?

c) The exact value of the integral is $\frac{3040}{3}$. Confirm that it lies between your two sums.

The Fundamental Theorem and Net Change

6. Evaluate each definite integral, showing the antiderivative you use.

a) $\displaystyle\int_0^2 \left(x^3 - 3x + 4\right)\, dx$

b) $\displaystyle\int_1^9 \frac{x+1}{\sqrt{x}}\, dx$

c) $\displaystyle\int_0^2 \frac{6x}{x^2+1}\, dx$

7. A coffee roaster produces x kilograms of coffee a week. Its marginal cost is $C'(x) = 9 + 0.0006x^2$ dollars per kilogram.

a) Find the increase in weekly cost when production rises from 100 to 200 kilograms.

b) Find the average value of the marginal cost over $100 \le x \le 200$, and say in one sentence what it means for the roaster.

Area Between Two Curves

8.

a) Find the area of the region enclosed by the parabolas $y = x^2 - 1$ and $y = 7 - x^2$.

b) Find the area of the region between $y = e^x$ and $y = e^{-x}$ for $0 \le x \le \ln 2$.

9. A print shop compares keeping its old press with leasing a new one. t years from now, the old press would cost $8 + 0.6t^2$ thousand dollars a year to run and the new one $5 + 1.2t$ thousand dollars a year. Ignore interest.

a) Show that the old press costs more to run at every time $t \ge 0$.

b) Find the total running-cost saving of the new press over the next 5 years, and say what region's area it is.

Consumer and Producer Surplus

10. In a regional market for wireless earbuds, the price-demand and price-supply functions are

$$p = D(x) = \frac{400}{x+4}, \qquad p = S(x) = 4 + 6x,$$

where x is the number of pairs in hundreds and p the price in dollars. At the equilibrium $(\bar{x}, \bar{p})$ the consumer surplus is $\int_0^{\bar{x}} (D(x) - \bar{p})\, dx$ and the producer surplus is $\int_0^{\bar{x}} (\bar{p} - S(x))\, dx$.

a) Find the equilibrium quantity and price.

b) Find the consumer surplus and the producer surplus, exactly and in dollars. Use $\ln 2.5 \approx 0.9162907$.

The Lorenz Curve and the Gini Index

11. The distribution of annual pay among the employees of a retail chain is described by the Lorenz curve

$$L(x) = 0.3x + 0.7x^2, \qquad 0 \le x \le 1,$$

meaning that the lowest-paid fraction x of employees receives the fraction $L(x)$ of the total payroll. The Gini index is $G = 2\int_0^1 (x - L(x))\,dx$.

a) Check that $L(0) = 0$, $L(1) = 1$ and $L(x) \le x$ on $[0, 1]$.

b) What percentage of the payroll goes to the lowest-paid half of employees?

c) Find the Gini index.

Synthesis — drawing on several topics in this unit

12. In a harbour-cruise market, the price-demand function is

$$p = D(x) = \frac{36}{\sqrt{2x+1}},$$

where x is the number of tickets sold in a season, in thousands, and p the ticket price in dollars. The market price settles where 4 thousand tickets are sold.

a) Find the market price.

b) Find the consumer surplus $\int_0^4 (D(x) - \bar{p})\,dx$ at that price, in dollars.